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TRANSIENT ANALYSIS OF POWER TRANSMISSION LINES USING THE DIGITA--ETC(U)
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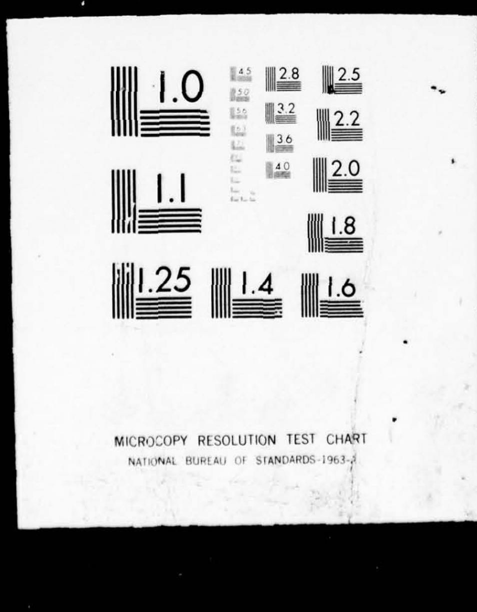

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TRANSIENT ANALYSIS OF POWER TRANSMISSION LINES

USING THE DIGITAL COMPUTER

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TRANSIENT ANALYSIS OF POWER TRANSMISSION LINES

USING THE DIGITAL COMPUTER

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Joel Douglas Benson, son of Julius Trenton and Mildred Eileen (Sims) Benson, was born on April 6, 1946, in Fairfield, Alabama. He attended Birmingham Public Schools and graduated from Ensley High School, Birmingham, in January, 1964. In January, 1964, he entered the University of Alabama and received the degree of Bachelor of Science in Electrical Engineering in 1969. During his undergraduate education, he was enrolled in the Cooperative Education Program, being employed by Southern Company Services, Inc., Birmingham. Upon completion of his undergraduate work, he entered the United States Air Force. After seven years in the Air Force he was selected to attend graduate school under a program with the Air Force Institute of Technology. In June, 1976, he entered the Graduate School at Auburn University. He married Linda, daughter of Claud and Dorothy (Whiten) Wilson in June, 1967. They have one daughter, Leisa Ann.

THESIS ABSTRACT

TRANSIENT ANALYSIS OF POWER TRANSMISSION LINES USING THE DIGITAL COMPUTER

Joel Douglas Benson

Master of Electrical Engineering, August 26, 1977
(B.S.E.E., University of Alabama, 1969)

97 Typed Pages

Directed by Charles A. Gross

A method is presented for modeling a transmission line for transient analysis study on the digital computer. The line is modeled as a finite number of lumped parameter sections. Each section is modeled in an equivalent section of resistors and current sources developed from solving the voltage and current equations by the trapezoidal rule for integration. The integration takes place over a period of time from a known state, t, to an unknown state, $t+\Delta t$. The time step, Δt, is taken to be the lossless travel time for the traveling wave to cross each section. The single phase lossless case is handled first, then losses are accounted for, and finally the three phase line is dealt with.

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I. INTRODUCTION

The need to study the transient phenomena on power transmission lines results from the high voltages experienced on the line due to normal but abrupt switching actions. These high voltages that appear are on an order of magnitude of two to three times the rated line voltage. Transients can be caused by other factors, such as atmospheric disturbance, but the majority is due to normal switching operations on the line. These transients usually last for only a few milliseconds [1], but insulators and other equipment can be permanently damaged. The study of transients on transmission lines has been underway for many years with the classical equations being well known. With the advent of the digital computer, methods are now available to solve the classical equations numerically and with a high degree of accuracy. This thesis deals with modeling the transmission line for the digital computer in order to solve for the transient voltages and currents that exist due to switching actions that can occur from energizing or deenergizing the line.

The solution to the classical transmission line equations is well known [1, 2, 3] and need not be presented

here. The nature of the solutions results in the concept of traveling waves on the transmission line. Since the transmission line parameters of resistance, inductance, capacitance, and conductance are distributed uniformly throughout the line, this provides the line with its wave carrying capability. It is much like any other physical continua, such as air and water, in this respect [1]. These traveling waves on the line are of two types--forward traveling and reverse traveling. The reverse traveling wave is a scaled version of the forward traveling wave. This scaling factor is called the reflection coefficient. Solutions have been very complicated except for the simplest cases and have typically dealt with a lossless line, i.e., resistance and conductance are assumed to be zero. One such solution utilizes the Bewley Lattice diagram which requires that the reflection coefficients for the sending and receiving ends be calculated [4].

Most all the work done in these solutions is for a single phase line. When three phase lines are studied, the concept of three phase is lost because of the transient phenomena. The line can be viewed as three separate phases by using a matrix transformation to decouple the phases. This approach is used in this thesis. In modeling the three phase line the earth return for the ground currents must be included, and in transient analysis this introduces the complex situation of handling the frequency dependency

of resistance and inductance of the ground mode [5]. This topic will be discussed later.

It has been mentioned that the transmission line is composed of uniformly distributed parameters. This thesis models the line as a finite number of sections each having lumped parameters, as shown in figure 1-1. The argument for this is that as the number of sections approaches infinity as their lengths become smaller and smaller, it approximates the distributed line. Initially, the line will be considered lossless with the lossy case being handled later.

In researching the literature a paper by Hermann W. Dommell, "Digital Computer Solution of Electromagnetic Transients in Single- and Multiphase Networks" [5], was listed as a source by nearly everyone who was working on the problem of digital solutions to transmission line transients. His method for solving transients is to handle the distributed parameters with a method called characteristics and the lumped parameters with the trapezoidal rule for integration. The method of characteristics is described more fully in a paper by F. H. Branin, Jr., "Transient Analysis of Lossless Transmission Lines" [6]. The inclusion of frequency dependent parameters in the problem is presented by Alan Budner in his paper, "Introduction of Frequency-Dependent Line Parameters into an Electromagnetic

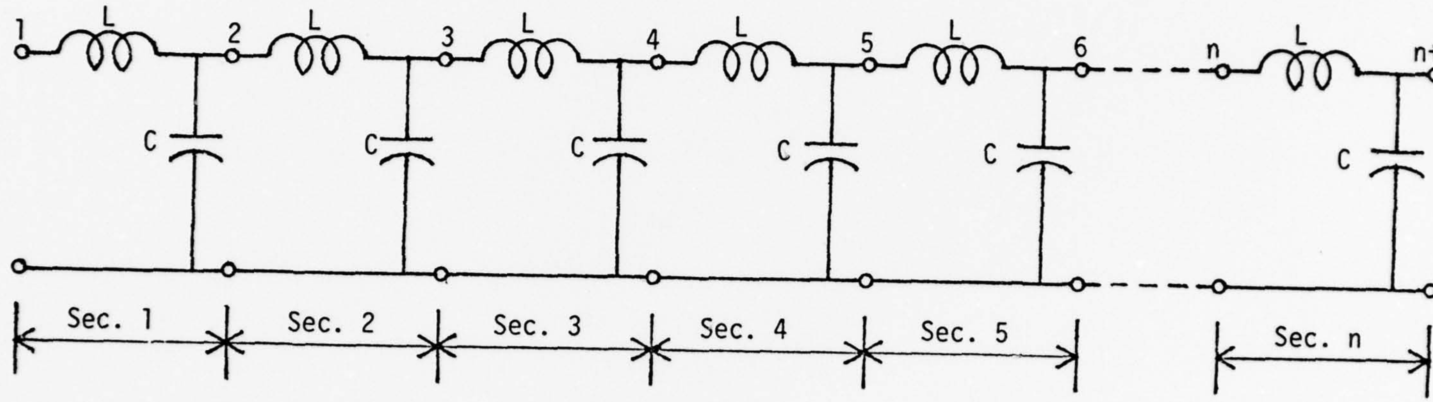

Figure 1-1. Transmission line represented by n-sections and n+1 nodes

Transient Program" [7]. This same problem is also discussed by J. K. Snelson in "Propagation of Traveling Waves on Transmission Lines--Frequency Dependent Parameters" [8]. S. C. Tripathy and N. D. Roa present a method in "A-Stable Numerical Integration Method for Transmission System Transients" [9] for handling nonlinear elements with a non-iterative technique.

The basis for this thesis is Dommell's work. A method will be developed to handle the transmission line on a digital computer for transient analysis. Lumped parameters for the transmission line will be dealt with exclusively. The lossless single-phase line will be developed first, then losses will be accounted for, and finally the three phase line will be analyzed.

II. THE SINGLE-PHASE LINE

Lossless Case

Figure 2-1 presents a typical section of the transmission line presented in figure 1-1. As the line is divided into sections, it will have n sections and n + 1 nodes. The development that follows will lend itself to digital computer techniques. Since the digital computer cannot give the entire listing of a transient on a transmission line [5], the development will be one that recognizes that it can give the results of computations at some time t + Δt where the results at time t are known. Referring to figure 2-1, the equation for the current through the inductor can be written as,

$$v_i - v_{i+1} = L \frac{di_i}{dt} \qquad (2\text{-}1a)$$

$$di_i = \frac{1}{L} (v_i - v_{i+1}) \, dt \qquad (2\text{-}1b)$$

Integrating from the known state, t, to an unknown state, t + Δt, using the trapezoidal rule for integration [10], gives

$$\int_t^{t+\Delta t} di_i = \frac{1}{L} \int_t^{t+\Delta t} (v_i - v_{i+1}) \, dt \qquad (2\text{-}1c)$$

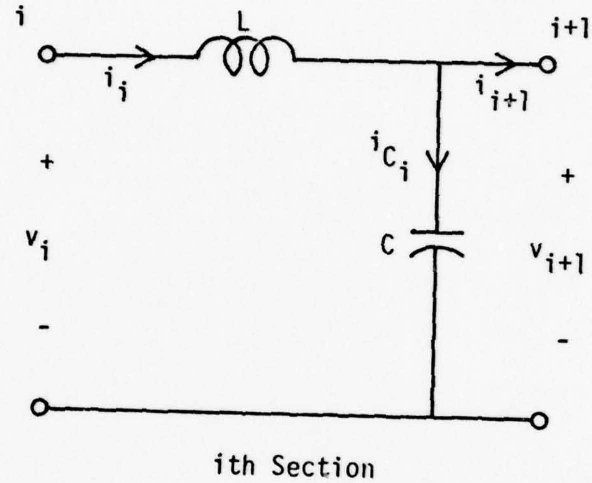

Figure 2-1. Typical lossless transmission line section

v_i at $t+\Delta t$ v_{i+1} at $t+\Delta t$

$$i_i(t+\Delta t) - i_i(t) = \frac{\Delta t}{2L} [v_i(t+\Delta t) - v_{i+1}(t+\Delta t)$$

v_i at t v_{i+1} at t

$$+ v_i(t) - v_{i+1}(t)] \qquad (2\text{-}1d)$$

$$i_i(t+\Delta t) = \frac{\Delta t}{2L} [v_i(t+\Delta t) - v_{i+1}(t+\Delta t)]$$

$$+ \frac{\Delta t}{2L} [v_i(t) - v_{i+1}(t)] + i_i(t) \qquad (2\text{-}1e)$$

In equation 2-1e the current at the ith node at $t+\Delta t$ is dependent on the difference in voltage at the i and i+1 nodes divided by an equivalent resistance ? between the two nodes, $\frac{2L}{\Delta t}$. The voltage and current in equation 2-1e at time t can be viewed as the past voltage and current, and therefore are known. Let,

$$I_i(t) = \frac{\Delta t}{2L} [v_i(t) - v_{i+1}(t)] + i_i(t) \qquad (2\text{-}1f)$$

These known values at time t will be viewed as a current source, $I_i(t)$.

Turning to the capacitor in the section, the current through it is given by,

$$i_{C_i} = C \frac{dv_{i+1}}{dt} \qquad (2\text{-}2a)$$

The current can also be expressed as,

$$i_{C_i} = i_i - i_{i+1} \qquad (2\text{-}2b)$$

Substituting equation 2-2b into 2-2a and rewriting,

$$dv_{i+1} = \frac{1}{C}(i_i - i_{i+1})dt \qquad (2\text{-}2c)$$

Using the trapezoidal rule and integrating from t to t+Δt,

$$\int_t^{t+\Delta t} dv_{i+1} = \frac{1}{C}\int_t^{t+\Delta t}(i_i - i_{i+1})dt \qquad (2\text{-}2d)$$

$$v_{i+1}(t+\Delta t) - v_{i+1}(t) = \frac{\Delta t}{2C}[i_i(t+\Delta t) - i_{i+1}(t+\Delta t)$$

$$+ i_i(t) - i_{i+1}(t)] \qquad (2\text{-}2e)$$

$$v_{i+1}(t+\Delta t) = \frac{\Delta t}{2C}[i_i(t+\Delta t) - i_{i+1}(t+\Delta t)]$$

$$+ \frac{\Delta t}{2C}[i_i(t) - i_{i+1}(t)]$$

$$+ v_{i+1}(t) \qquad (2\text{-}2f)$$

The known values in equation 2-2f now appear as a voltage source. Let,

$$V_{i+1}(t) = \frac{\Delta t}{2C}[i_i(t) - i_{i+1}(t)] + v_{i+1}(t) \qquad (2\text{-}2g)$$

The equivalent circuit for the line section is shown in figure 2-2. Since the network now lends itself to general nodal analysis, it is desirable to transform the voltage source to a current source. Using Norton's Theorem to accomplish this, the resulting circuit is shown in figure 2-3.

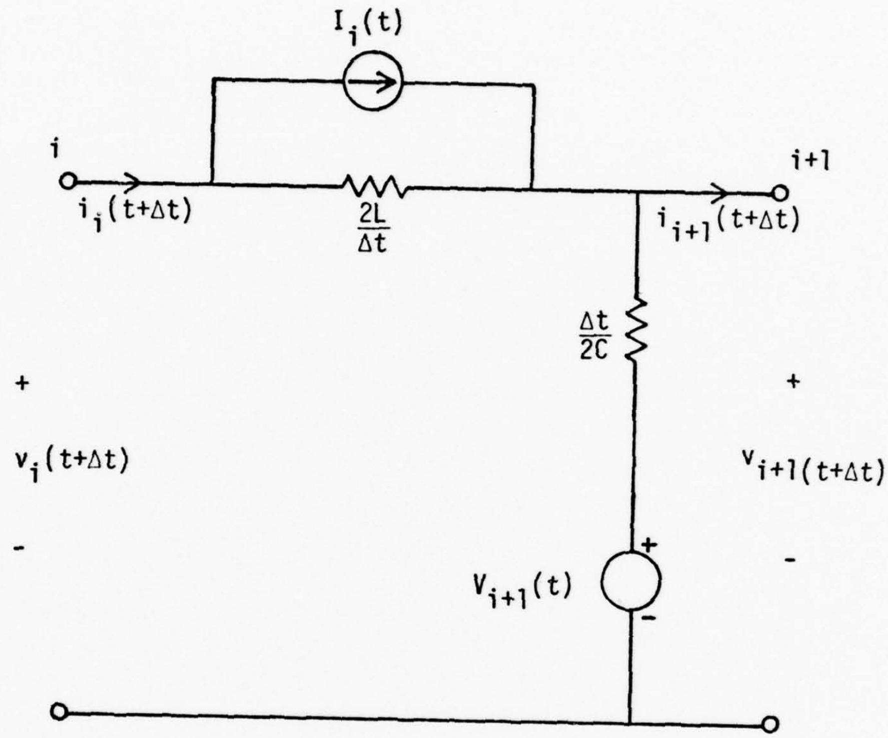

Figure 2-2. Typical lossless transmission line equivalent section

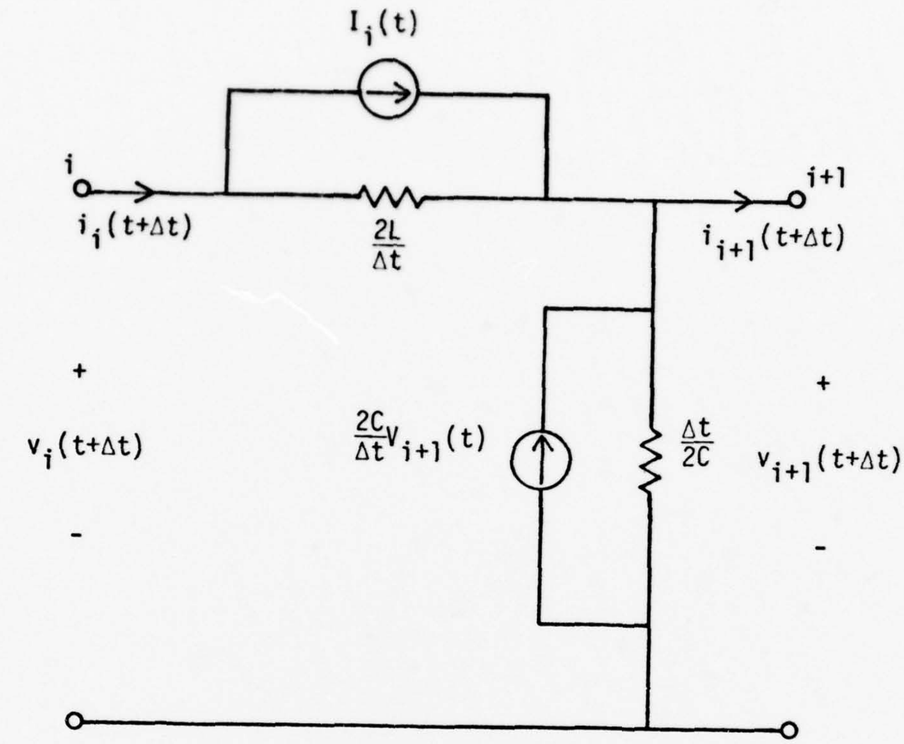

Figure 2-3. Typical transmission line equivalent section with all current sources

Starting with the typical section of the transmission line composed of inductance and capacitance, the equivalent circuit is one composed of resistive elements and current sources. This allows analysis of each section to be done by nodal techniques and without solving differential equations. The entire equivalent transmission line is shown in figure 2-4. For nodal analysis, it is more convenient to deal with conductance than resistance. In figure 2-4.

$$G_s = \frac{\Delta t}{2L} \quad \text{and,} \tag{2-3}$$

$$G_p = \frac{2C}{\Delta t} \tag{2-4}$$

Writing a matrix equation for the entire line using conventional nodal analysis,

$$[Y]\tilde{v} = \tilde{C} \tag{2-5a}$$

where, excluding the end nodes,

$$y_{ij} = \begin{cases} 2G_s + G_p, & i = j = 2,3, \ldots, n \\ -G_s, & i = j\pm 1 \\ 0, & i = j\pm 2,3, \ldots, n \end{cases} \tag{2-5b}$$

The general entry for $\tilde{v}$ is $v_i(t+\Delta t)$ and for $\tilde{C}$, a current vector, is given by,

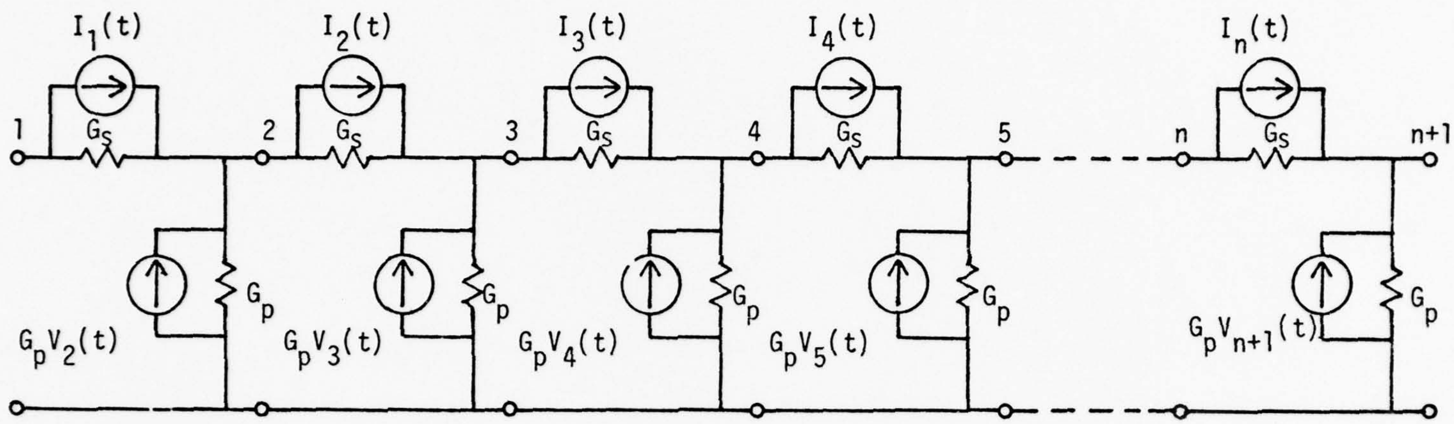

Figure 2-4. Lossless transmission line represented by n equivalent sections

$$c_i = I_{i-1}(t) + G_p \cdot V_i(t) - I_i(t),\ i = 2, \ldots, n \qquad (2\text{-}5c)$$

where, again excluding the end nodes,

$$I_{i-1}(t) = G_s[v_{i-1}(t) - v_i(t)] + i_{i-1}(t) \qquad (2\text{-}5d)$$

$$G_p V_i(t) = i_{i-1}(t) - i_i(t) + G_p v_i(t) \qquad (2\text{-}5e)$$

$$I_i(t) = G_s[v_i(t) - v_{i+1}(t)] + i_i(t) \qquad (2\text{-}5f)$$

The solution to equation 2-5a is given by,

$$\tilde{v} = [Y]^{-1}\tilde{C} \qquad (2\text{-}5g)$$

A method for inverting the Y-matrix, which is a sparse matrix, is given in Appendix A.

The source for the line was chosen to be a current source with shunted inductance, capacitance, and resistance as shown in figure 2-5. Each element of the source will be handled separately in order to develop a model compatible with the line model.

For the inductor,

$$v_1 = L_s \frac{di_{L_s}}{dt} \qquad (2\text{-}6a)$$

Using the trapezoidal rule and integrating from t to t+Δt,

$$\int_t^{t+\Delta t} di_{L_s} = \frac{1}{L_s}\int_t^{t+\Delta t} v_1 dt \qquad (2\text{-}6b)$$

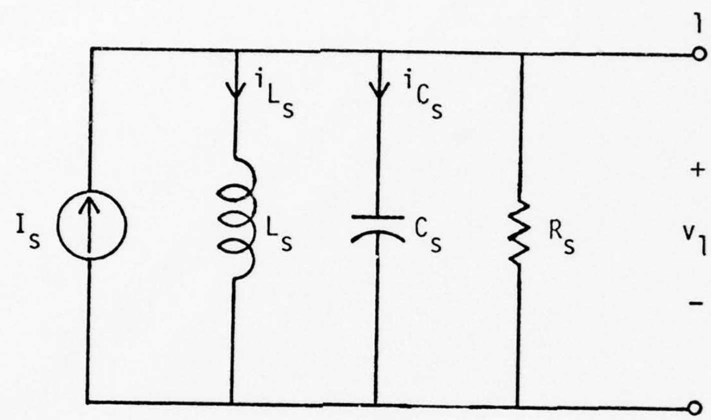

Figure 2-5. Source model

$$i_{L_s}(t+\Delta t) - i_{L_s}(t) = \frac{\Delta t}{2L_s}\left[v_1(t+\Delta t) + v_1(t)\right] \qquad (2\text{-}6c)$$

$$i_{L_s}(t+\Delta t) = \frac{\Delta t}{2L_s} v_1(t+\Delta t) + \frac{\Delta t}{2L_s} v_1(t) + i_{L_s}(t) \qquad (2\text{-}6d)$$

let,

$$I_{L_s}(t) = -\frac{\Delta t}{2L_s} v_1(t) - i_{L_s}(t) \qquad (2\text{-}6e)$$

then,

$$i_{L_s}(t+\Delta t) = \frac{\Delta t}{2L_s} v_1(t+\Delta t) - I_{L_s}(t) \qquad (2\text{-}6f)$$

Similarly, for the capacitor,

$$i_{C_s} = C_s \frac{dv_1}{dt} \qquad (2\text{-}7a)$$

$$i_{C_s}(t+\Delta t) = \frac{2C_s}{\Delta t} v_1(t+\Delta t) - \frac{2C_s}{\Delta t} v_1(t) - i_{C_s}(t) \qquad (2\text{-}7b)$$

let,

$$I_{C_s}(t) = \frac{2C_s}{\Delta t} v_1(t) + i_{C_s}(t) \qquad (2\text{-}7c)$$

then,

$$i_{C_s}(t+\Delta t) = \frac{2C_s}{\Delta t} v_1(t+\Delta t) - I_{C_s}(t) \qquad (2\text{-}7d)$$

Since the resistor does not contribute a current source in modeling, it remains unchanged. The equivalent source

model is shown in figure 2-6. In order to simplify the model and make it more compatible with the line and also for nodal analysis, the current sources are combined and resistive elements are also combined but as conductances. The final result is shown in figure 2-7, with,

$$I_g(t) = I_s(t) + I_{L_s}(t) + I_{L_s}(t) \tag{2-8}$$

and,

$$G_g = \frac{\Delta t}{2L_s} + \frac{2C_s}{\Delta t} + \frac{1}{R_s} \tag{2-9}$$

The receiving end termination can also be modeled in a general circuit of shunt resistance, inductance and capacitance. Following the same argument as before in the source, the receiving end circuit for a generalized load is shown in figure 2-8. As before,

$$G_L = \frac{\Delta t}{2L_L} + \frac{2C_L}{\Delta t} + \frac{1}{R_L} \tag{2-10}$$

$$I_L(t) = I_{C_L}(t) + I_{L_L}(t) \tag{2-11a}$$

where,

$$I_{C_L}(t) = \frac{2C_L}{\Delta t} v_{n+1}(t) + i_{C_L}(t) \tag{2-11b}$$

$$I_{L_L}(t) = -\frac{\Delta t}{2L_L} v_{n+1}(t) - i_{L_L}(t) \tag{2-11c}$$

Returning to the nodal equation,

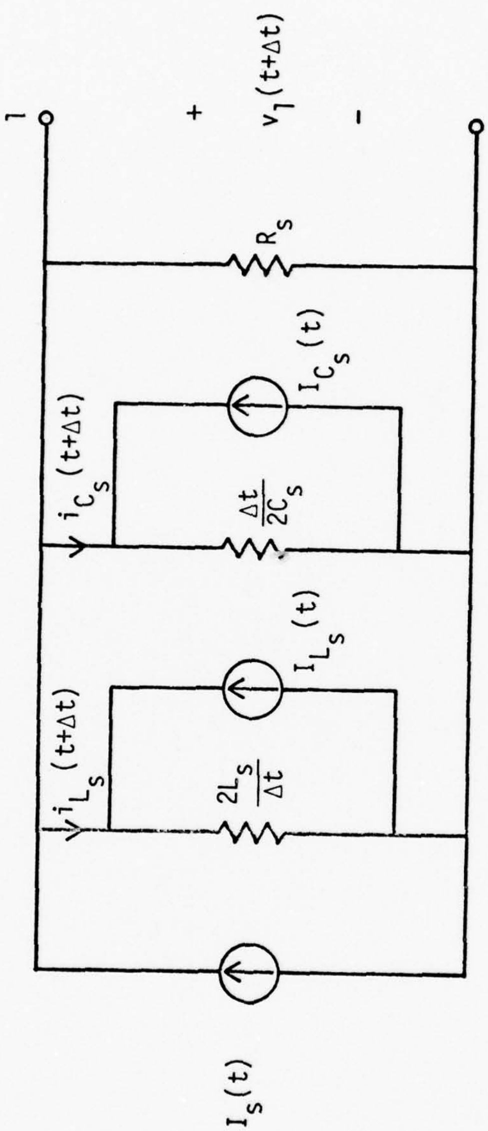

Figure 2-6. Equivalent source model

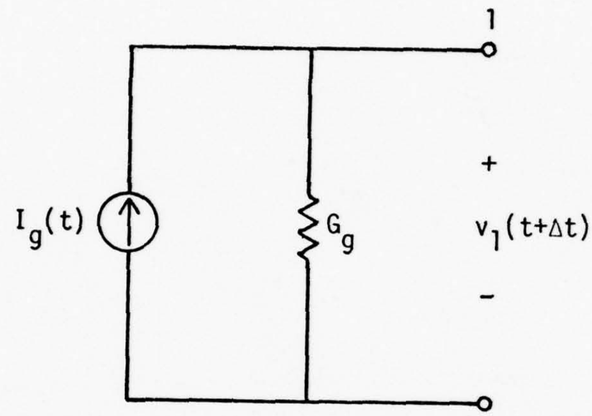

Figure 2-7. Combined equivalent source model

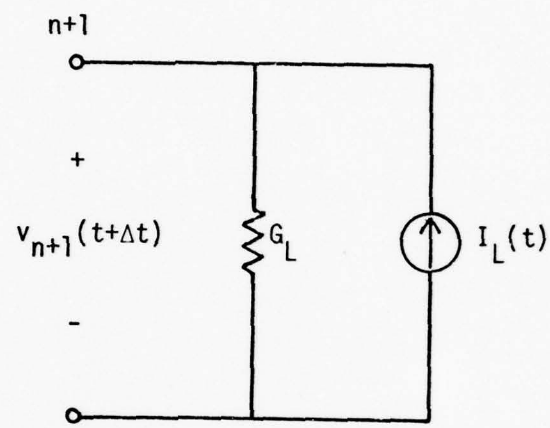

Figure 2-8. Equivalent receiving-end model

$$[Y]\tilde{v} = \tilde{C} \tag{2-5a}$$

the entries of the Y-matrix and C-vector can now be completed. For the Y-matrix,

$$y_{11} = G_g + G_s \tag{2-5h}$$

where,

$$G_g = \frac{\Delta t}{2L_s} + \frac{2C_s}{\Delta t} + \frac{1}{R_s} \tag{2-9}$$

$$G_s = \frac{\Delta t}{2L} \tag{2-3}$$

and,

$$y_{n+1,n+1} = G_s + G_p + G_L \tag{2-5i}$$

where,

$$G_p = \frac{2C}{\Delta t} \tag{2-4}$$

For the C-vector,

$$c_1 = I_g(t) - I_1(t) \tag{2-5j}$$

where,

$$I_g(t) = I_s(t) + I_{L_s}(t) + I_{C_s}(t) \tag{2-8}$$

$$I_1(t) = G_s[v_1(t) - v_2(t)] + i_1(t) \tag{2-5f}$$

and,

$$c_{n+1} = I_n(t) + G_p V_{n+1}(t) + I_L(t) \tag{2-5k}$$

where,

$$I_n(t) = G_s[v_n(t) - v_{n+1}(t)] + i_n(t) \qquad (2\text{-}5f)$$

$$G_p V_{n+1}(t) = i_n(t) - i_{n+1}(t) + G_p v_{n+1}(t) \qquad (2\text{-}5e)$$

$$I_L(t) = i_{C_L}(t) + I_{L_L}(t) \qquad (2\text{-}11a)$$

Lossless Case Examples

An example problem was chosen to apply to the preceding development. Computer input data for the cases tested, 2, 10, and 40 section lines, are shown in figure 2-9, with only the number of sections changing for each example. Data were chosen to facilitate manual calculations. The velocity of wave propagation on a lossless line is given by [11],

$$\text{velocity} = \frac{1}{\sqrt{LC}} \qquad (2\text{-}12)$$

and the characteristic impedance is given by [11],

$$Z_o = \sqrt{\frac{L}{C}} \qquad (2\text{-}13)$$

For the example chosen, both of these are one and the sending and receiving ends are terminated in the characteristic impedance. With this configuration, there should be no reflected voltages along the line and with the one amp current source, voltage and current should stabilize at the same value, 0.5. The delta t chosen for the equations

LINE DATA

LINE INDUCTANCE (H/M)	LINE CAPACITANCE (F/M)	LINE LENGTH (M)	NUMBER SECTIONS
1.000	1.000	1.000	2, 10, or 40

SOURCE DATA

SOURCE CURRENT (AMPS)	SOURCE GAMMA (1/L)	SOURCE CAPACITANCE (F)	SOURCE CONDUCTANCE (MHOS)
1.000	0.000	0.000	1.000

LOAD DATA

LOAD GAMMA (1/L)	LOAD CAPACITANCE (F)	LOAD CONDUCTANCE (MHOS)
0.000	0.000	1.000

Figure 2-9. Computer input for 2, 10, and 40 section line examples

corresponds to the travel time for the line section. Results for the three cases tested, 2, 10, and 40 sections, are shown in figures 2-10, 2-11, and 2-12, respectively. The plots show voltage as function of position at a time that corresponds to the wave traveling halfway down the line. The theoretical wave shapes are shown as dashed lines. As might be expected, the 40 section line exhibited a waveform that more closely approximated the theoretical, and is more oscillatory in nature than the other cases. The calculations for the three cases were allowed to continue for a time period until they stabilized and these results are shown in figures 2-13, 2-14, and 2-15.

In order to see the effect of the size of delta t, a smaller delta t than for any previous case was chosen, $\Delta t = .01$, and the three cases run again. These results are shown in figures 2-16, 2-17, and 2-18 for a travel time of halfway down the line. When compared with figures 2-10, 2-11, and 2-12, respectively, the wave shapes do not appear very different, except that they are generally steeper at the leading edge. To further check its effect, runs were made with the 40 section line for varying delta t's, larger and smaller than the travel time for each section. These results are shown in figures 2-19, 2-20, and 2-21. The smallest delta t chosen was 0.001, figure 2-21, and results do not vary appreciably from that of 0.005 in figure 2-20. These results confirm Dommell's results [5] that changing delta t tends to change the phase

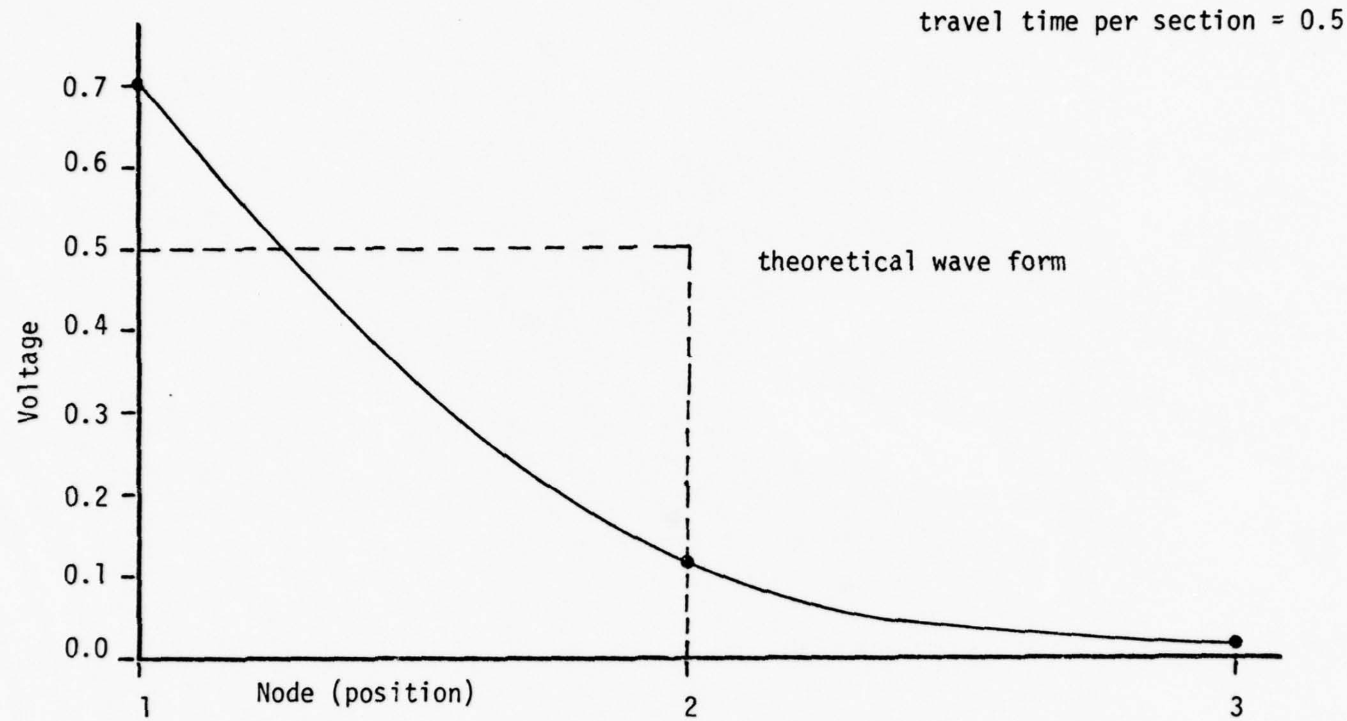

Figure 2-10. Voltage versus position for a 2 section line, $\Delta t = 0.5$, $t = 0.5$

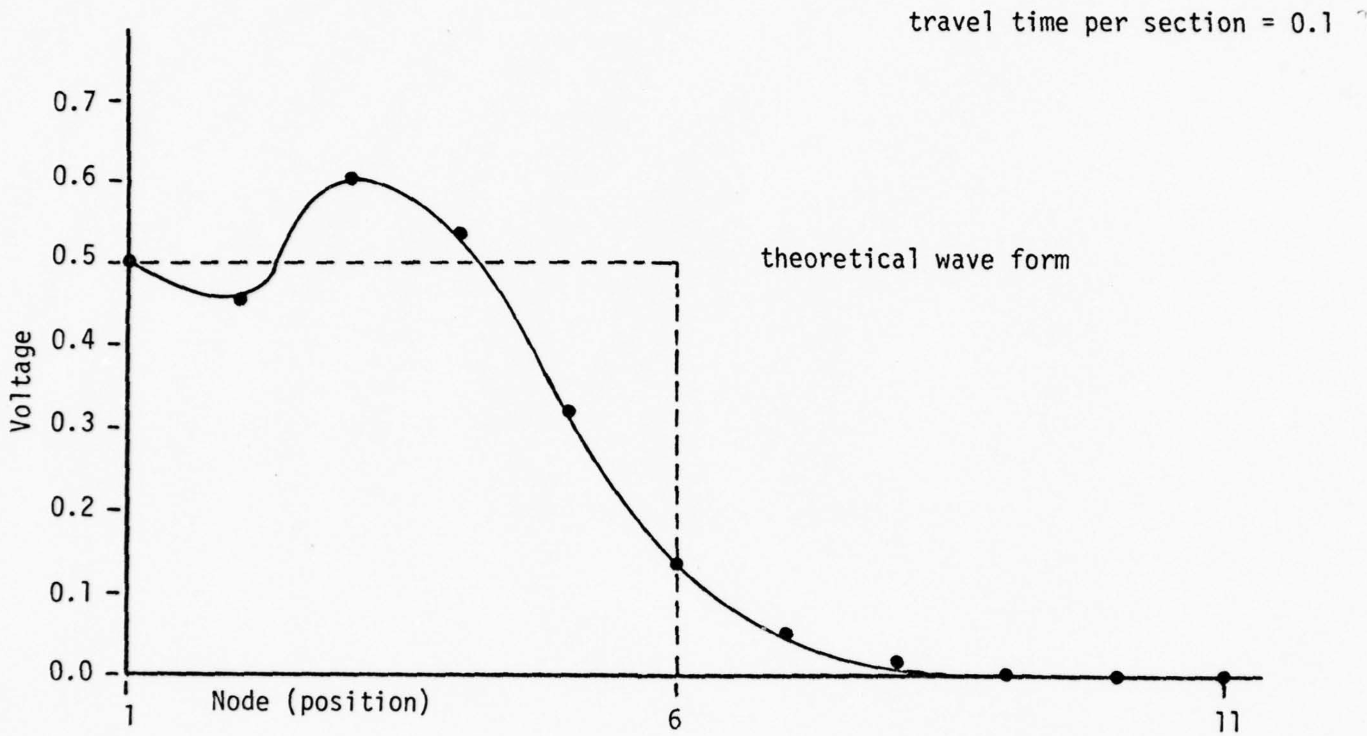

Figure 2-11. Voltage versus position for a 10 section line, $\Delta t = 0.1$, $t = 0.5$

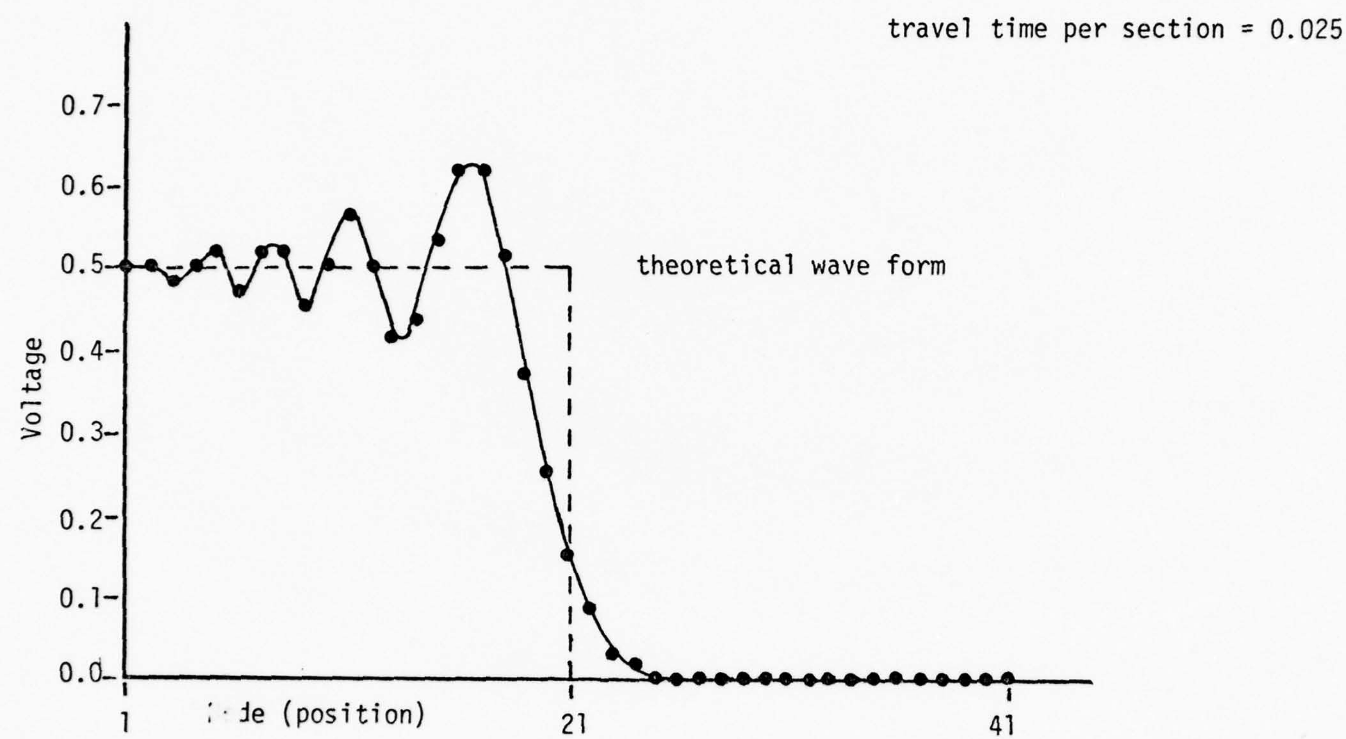

Figure 2-12. Voltage versus position for a 40 section line, $\Delta t = 0.025$, $t = 0.5$

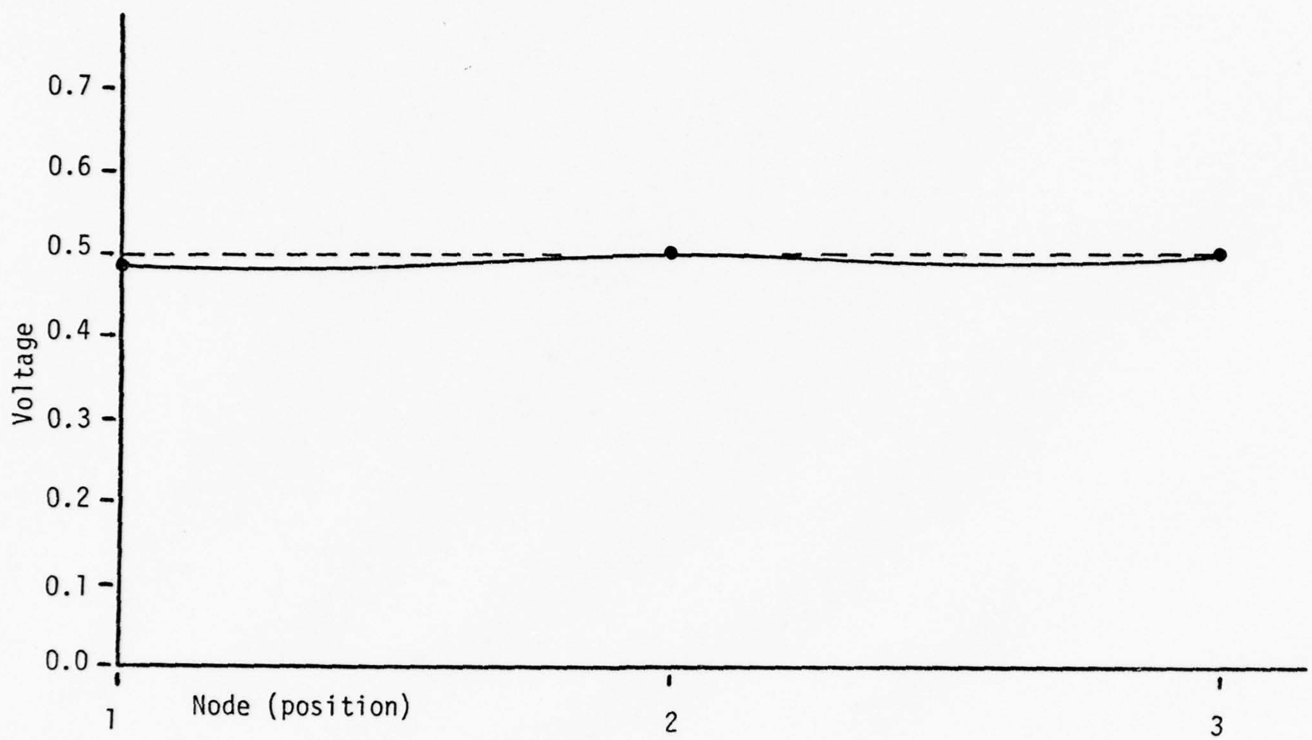

Figure 2-13. Voltage versus position for a 2 section line, $\Delta t = 0.5$, $t = 8.0$

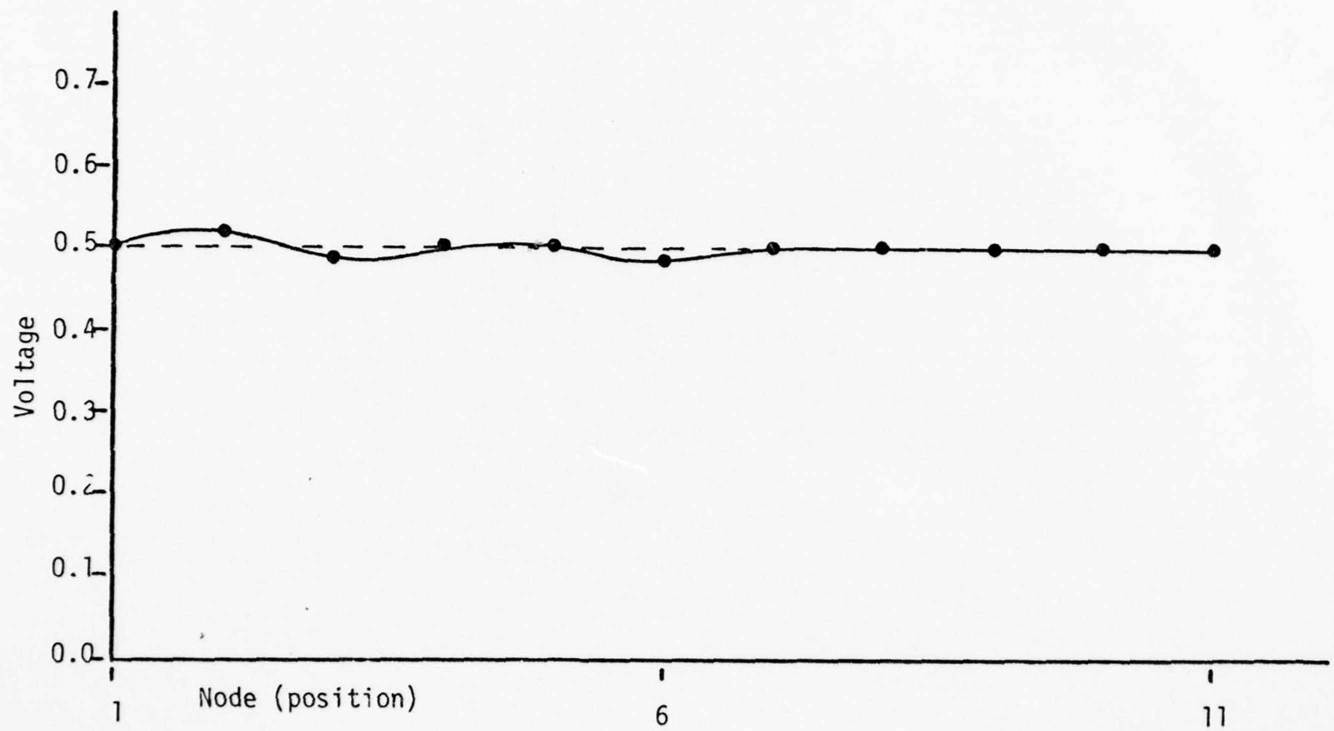

Figure 2-14. Voltage versus position for a 10 section line, $\Delta t = 0.1$, $t = 8.0$

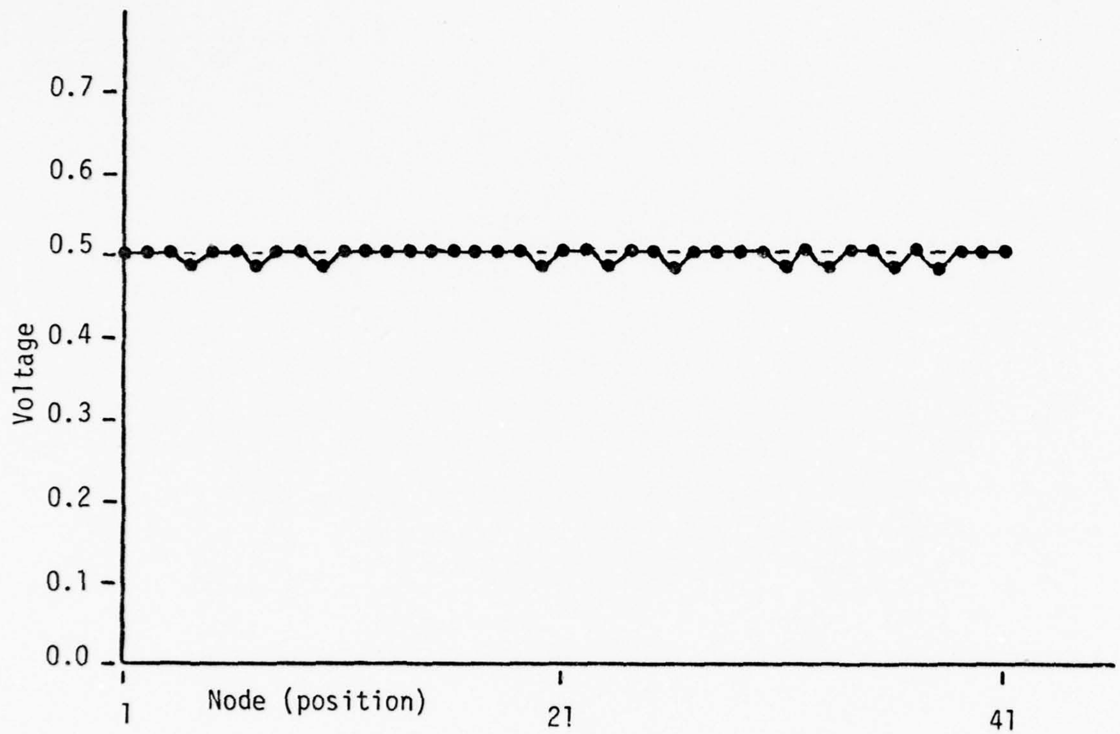

Figure 2-15. Voltage versus position for a 40 section, $\Delta t = 0.025$, $t = 8.0$

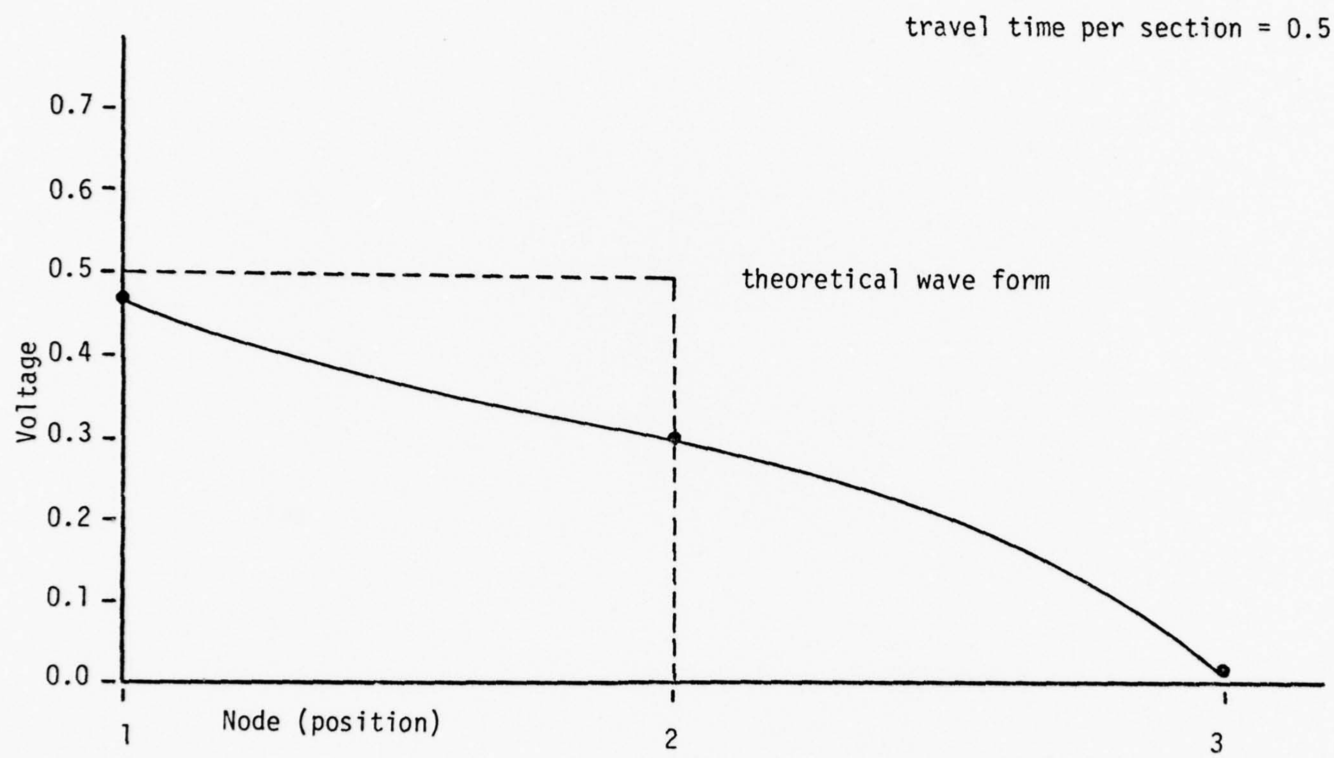

Figure 2-16. Voltage versus position for a 2 section line, $\Delta t = 0.01$, $t = 0.5$

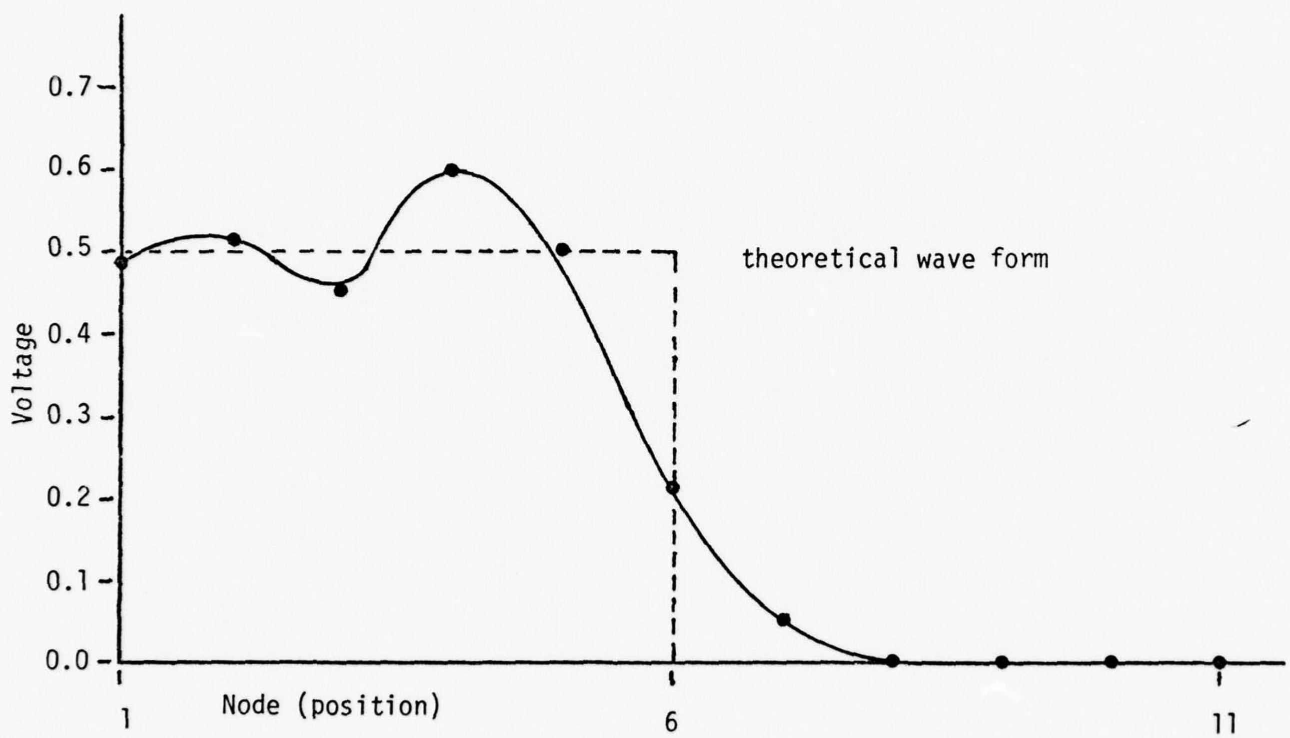

Figure 2-17. Voltage versus position for a 10 section line, $\Delta t = .01$, $t = 0.5$

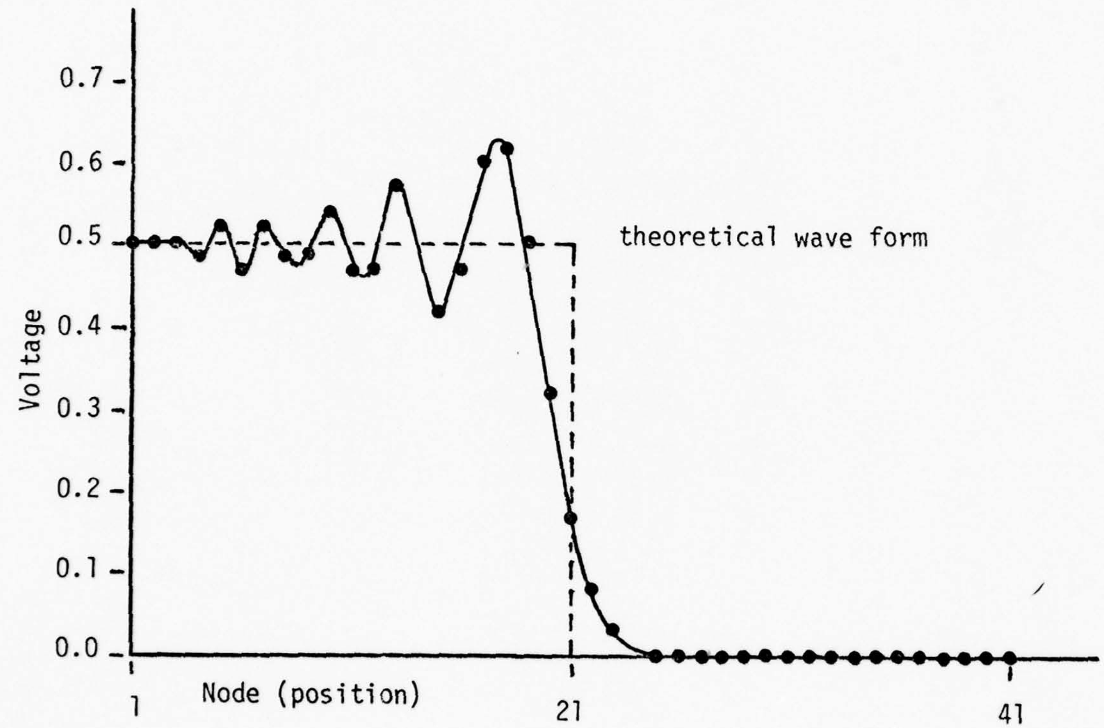

Figure 2-18. Voltage versus position for a 40 section line, $\Delta t = .01$, $t = 0.5$

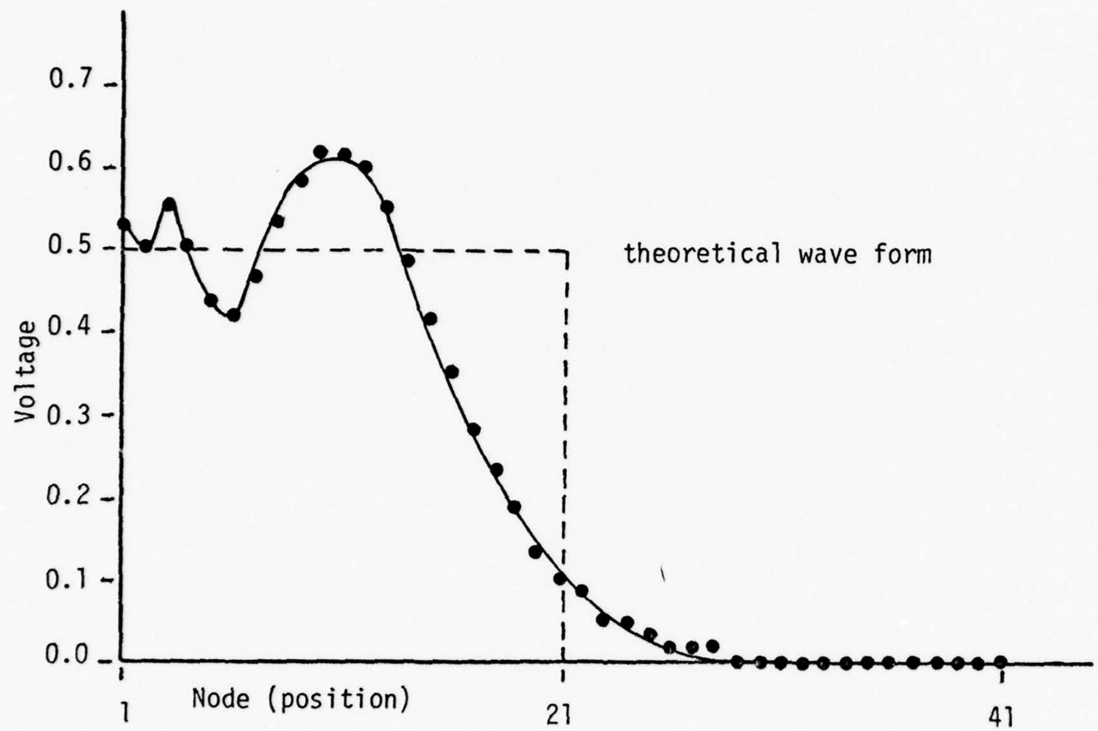

Figure 2-19. Voltage versus position for a 40 section line, $\Delta t = .1$, $t = 0.5$

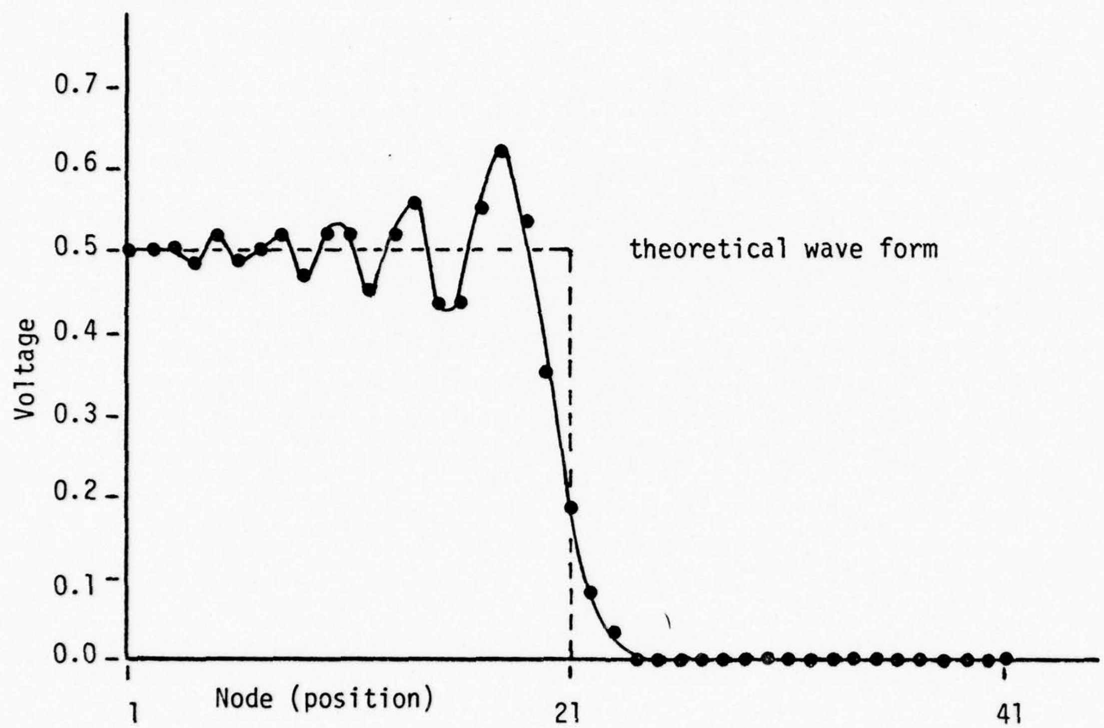

Figure 2-20. Voltage versus position for a 40 section line, Δt = .005, t = 0.5

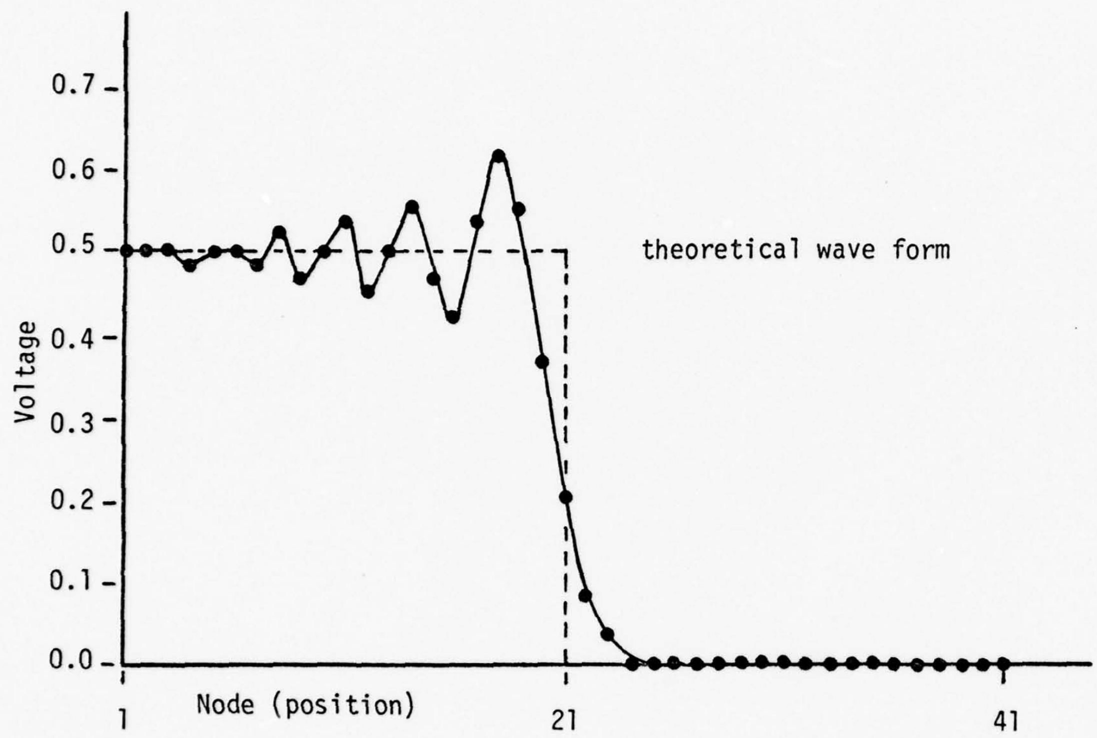

Figure 2-21. Voltage versus position for a 40 section line, Δt = .001, t = 0.5

position of the high frequency oscillations but not their amplitudes. This does not make the choice of delta t critical and for the remainder of this thesis delta t will be chosen to be the lossless travel time for the line section. The length of the line section was determined from the fact that most delta t's are in the neighborhood of 50 microseconds. Since the velocity of a wave on a lossless line is approximately the speed of light, the wave travels approximately 15 kilometers (9.3 miles) in 50 microseconds. This figure of 15 kilometers is used to determine, to the nearest whole number, the number of sections needed to represent the line under consideration.

Lossy Case

In the lossless development, the series resistance of the line was ignored. However, the same arguments can be made with resistance included. The typical line section is shown in figure 2-22. Writing a voltage equation for the section,

$$v_i = L\,\frac{di_i}{dt} + Ri_i + v_{i+1} \qquad \text{(2-14a)}$$

rewriting,

$$di_i = \frac{1}{L}(v_i - Ri_i - v_{i+1}) \qquad \text{(2-14b)}$$

Using the trapezoidal rule and integrating from t to $t+\Delta t$,

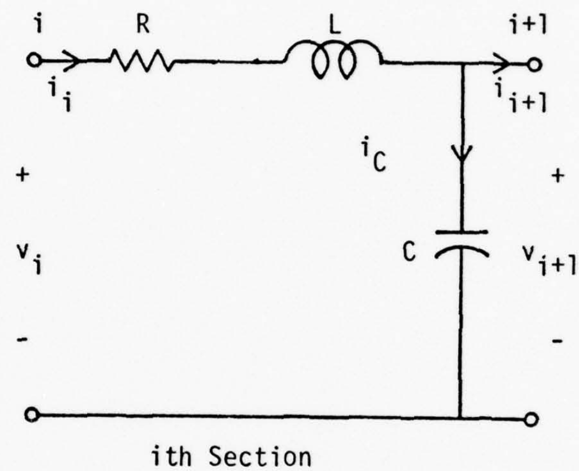

Figure 2-22. Typical lossy transmission line section

$$\int_{t}^{t+\Delta t} di_i = \frac{1}{L}\int_{t}^{t+\Delta t} (v_i - Ri_i - v_{i+1})dt \tag{2-14c}$$

$$\begin{aligned} i_i(t+\Delta t) - i_i(t) = \frac{\Delta t}{2L}\{v_i(t+\Delta t) + v_i(t) - R[i_i(t+\Delta t) \\ + i_i(t)] - [v_{i+1}(t+\Delta t) \\ + v_{i+1}(t)]\} \end{aligned} \tag{2-14d}$$

$$\begin{aligned} [1 + \frac{R\Delta t}{2L}]i_i(t+\Delta t) = \frac{\Delta t}{2L}[v_i(t+\Delta t) - v_{i+1}(t+\Delta t)] \\ + \frac{\Delta t}{2L}[v_i(t) - v_{i+1}(t)] \\ + [1 - \frac{R\Delta t}{2L}]i_i(t) \end{aligned} \tag{2-14e}$$

$$\begin{aligned} i_i(t+\Delta t) = \left[\frac{1}{\frac{2L}{\Delta t} + R}\right][v_i(t+\Delta t) - v_{i+1}(t+\Delta t)] \\ + \left[\frac{1}{\frac{2L}{\Delta t} + R}\right][v_i(t) - v_{i+1}(t)] \\ + \left[\frac{\frac{2L}{\Delta t} - R}{\frac{2L}{\Delta t} + R}\right] i_i(t) \end{aligned} \tag{2-14f}$$

Let,

$$\begin{aligned} I_i(t) = \left[\frac{1}{\frac{2L}{\Delta t} + R}\right][v_i(t) - v_{i+1}(t)] \\ + \left[\frac{\frac{2L}{\Delta t} - R}{\frac{2L}{\Delta t} + R}\right] i_i(t) \end{aligned} \tag{2-14g}$$

The model for the remainder of the section remains the same as the lossless model. The lossy circuit model appears now as in figure 2-23, where,

$$v_{i+1}(t) = \frac{\Delta t}{2C}\,[i_i(t) - i_{i+1}(t)] + v_{i+1}(t) \qquad (2\text{-}2g)$$

If resistance is set to zero in this model, then it reduces to the lossless case. The same nodal equations can be written as in the lossless case with the appropriate changes made to the Y-matrix and C-vector.

Lossy Example

An example was chosen from Dommell's work [5] to compare results of the two programs. The line data is listed below.

line length	=	320 miles
R	=	0.0376 ohms/mi
L	=	1.52 mH/mi
C	=	0.0143 μF/mi
line termination	=	0.1 H
source	=	10 V. step

The data was scaled in order to input it to the computer. The scaling method is described in Appendix B. Since Dommell's program used 32 sections to represent the line, the same number was used in this example. The results are shown in figure 2-24. The agreement between the two program results

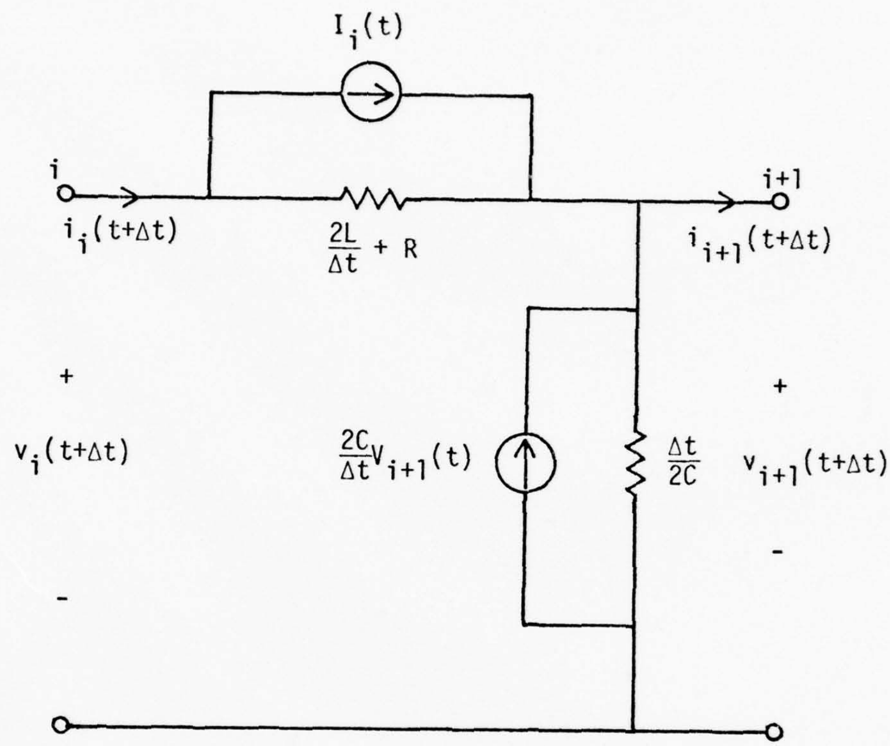

Figure 2-23. Typical lossy transmission line equivalent section

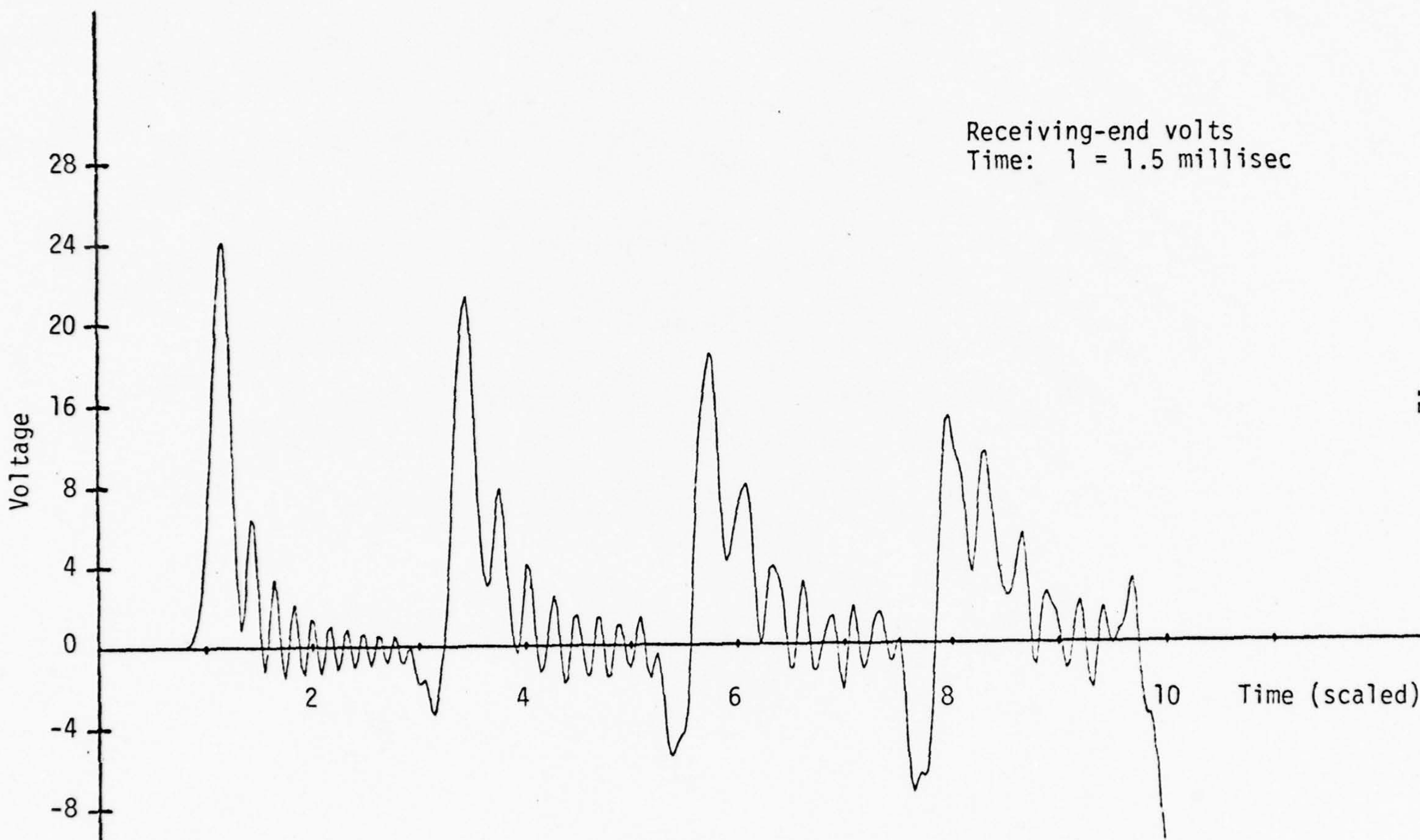

Figure 2-24. Lossy example

is very good. Wave shape is almost identical with slight variation in amplitudes. This possibly is due to the fact that Dommell's method for handling resistance is different from this thesis' method. Also, it is not precisely known how Dommell handled end effects.

Modeling a Sinusoidal Voltage Source

Modeling an ideal, non-time varying voltage source as a current source, as in the previous example, proved to be straight forward. Technically, the Norton Equivalent of an ideal voltage source is an infinite current source shunted by a zero resistance. For the computer program developed for this analysis, the ideal voltage source was modeled as a very large current source shunted by a very small resistance. The values were chosen such that the open circuit voltage equaled that of the voltage source. This method also works for a sinusoidal source but it is modified slightly.

When a sinusoidal source is used as a model for energizing a transmission line it is usually modeled as a generator with a series impedance made up of inductance and and resistance. Choosing a model for the network that energizes a transmission line is a non-trivial task, but it is not the subject of this thesis. In this thesis, a sinusoidal voltage source with series impedance is modeled in the following way. The source itself is always modeled as an ideal

voltage source, while the series impedance is added as a new section to the beginning of the line. This new section is handled as was the lossy section handled earlier in this chapter (Equations 2-14a-g). This method increases the number of sections and nodes by one. This development becomes more important when three phase lines are encountered. The equivalent model for a general sinusoidal voltage is shown in figure 2-25.

When this source model is used to energize the line, the Y-matrix and C-vector are modified as follow. Referring to figure 2-25, let

$$G_{g_s} = 1/[(2L_s/\Delta t) + R_s] \qquad (2\text{-}15)$$

Equation 2-9 now becomes

$$G_g = 10^6 \qquad (2\text{-}9')$$

Then

$$y_{0,0} = G_g + G_{g_s} \qquad (2\text{-}5\ell)$$

and,

$$y_{0,1} = y_{1,0} = -G_{g_s} \qquad (2\text{-}5m)$$

Equation 2-5h now becomes

$$y_{11} = G_{g_s} + G_s \qquad (2\text{-}5h')$$

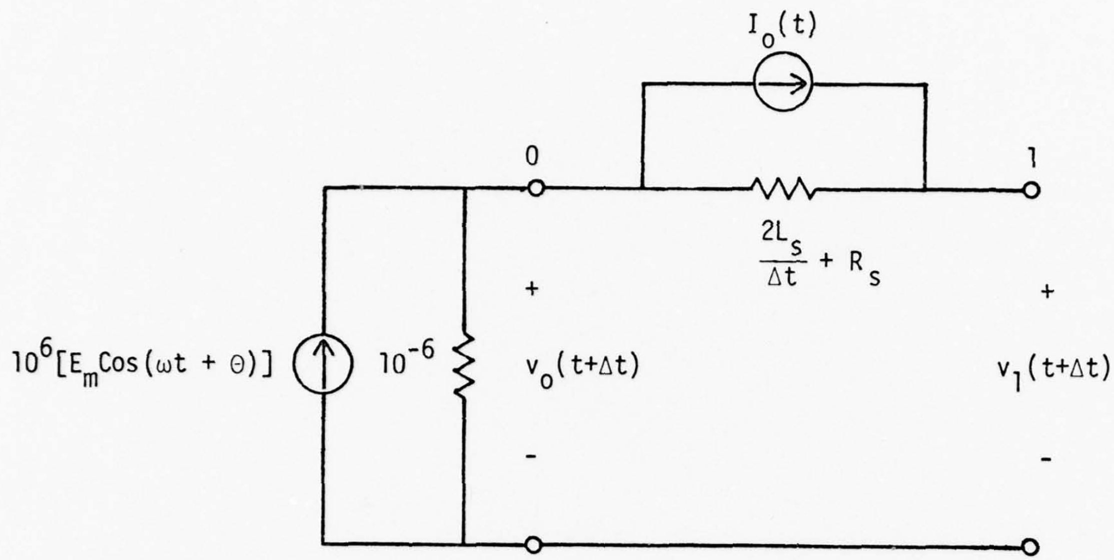

Figure 2-25. Equivalent model for a general sinusoidal voltage source

and equation 2-5b becomes

$$y_{ij} = \begin{cases} 2G_s + G_p & , \quad i = j \quad , \quad j = 2,3, \ldots, n \\ -G_s & , \quad i = j \pm 1, \quad j = 2,3, \ldots, n \\ 0 & , \quad i = j \pm 2,3, \ldots, n \end{cases} \qquad (2\text{-}5b')$$

except as noted. Equation 2-5i remains unchanged. For the C-vector changes, let

$$I_0(t) = \left[\frac{1}{\frac{2L_s}{\Delta t} + R_s} \right][v_0(t) - v_1(t)]$$

$$+ \left[\frac{\frac{2L_s}{\Delta t} - R_s}{\frac{2L_s}{\Delta t} + R_s} \right] i_0(t) \qquad (2\text{-}5n)$$

Equation 2-8 now becomes

$$I_g(t) = 10^6[E_m \text{Cos}(\omega t + \Theta)] \qquad (2\text{-}8')$$

then,

$$c_0 = I_g(t) - I_0(t) \qquad (2\text{-}5p)$$

Equation 2-5j now becomes

$$c_1 = I_0(t) - I_1(t) \qquad (2\text{-}5j')$$

and equation 2-5c is valid except the subscript i now starts at 3 instead of 2.

III. THE THREE PHASE LINE

Three Phase Model

In dealing with the three phase line under transient conditions, it is desirable to analyze each phase separately. To accomplish this, the phases must be decoupled because of the mutual inductances that exist between them. This can be done with a similarity transformation matrix, known as a modal transformation matrix, that diagonalizes the line impedance and admittance matrices [2]. In Dommell's paper [6], he introduces a modal transformation matrix [T] for a three phase line as,

$$[T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \tag{3-1}$$

with

$$[T]^{-1} = 1/3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

This matrix is only valid for a completely transposed line. There are other modal matrices which are used on all types of lines but they will produce off-diagonal elements in the

transformed matrices. These off-diagonal elements are small when compared with the diagonal elements, and are generally ignored. This is strictly true assuming a totally transposed line, which will not produce off-diagonal elements when transformed. In this work, off-diagonal elements will be ignored. This allows the sequence values to be substituted for the modal values since they are equal. Specifically

$$R_o = \mathrm{Re}[z_0] \qquad \text{ohm/m} \qquad (3\text{-}2a)$$

$$L_o = \frac{\mathrm{Im}[z_0]}{\omega} \qquad \text{H/m} \qquad (3\text{-}2b)$$

$$C_o = \frac{\mathrm{Im}[y_0]}{\omega} \qquad \text{F/m} \qquad (3\text{-}2c)$$

$$R_\alpha = R_\beta = \mathrm{Re}[z_1] \qquad \text{ohm/m} \qquad (3\text{-}2d)$$

$$L_\alpha = L_\beta = \frac{\mathrm{Im}[z_1]}{\omega} \qquad \text{H/m} \qquad (3\text{-}2e)$$

$$C_\alpha = C_\beta = \frac{\mathrm{Im}[y_1]}{\omega} \qquad \text{F/m} \qquad (3\text{-}2f)$$

The phase voltages and currents are defined in terms of the modal values as follows:

$$\tilde{v}_{abc} = [T]\tilde{v}_{o\alpha\beta} \tag{3-3}$$

and,

$$\tilde{i}_{abc} = [T]\tilde{i}_{o\alpha\beta} \tag{3-4}$$

With the line defined now with its modal values, the problem reverts back to the single phase case as described in Chapter 2. Each mode will be treated as the equivalent line in figure 2-4, except with losses. After each mode is solved, essentially three single phase problems, equations 3-3 and 3-4 will be used to find the phase values.

The end effects for the three phase line are essentially handled as in the single phase case. The three phase network energizing the line will be assumed to be a three phase voltage source with series impedance. The voltage source will be handled as described in Chapter 2.

<u>Three Phase Example</u>

A three phase example problem was chosen from work done by Southern Company Services, Inc., with their transient program "Surge." The line and system data are listed below.

System: 345 KV, 100 MVA, 50 Hz

Source data

positive sequence voltage (p.u.):	$1.0011\angle 0°$ (line to neutral peak value)
impedance (p.u.):	0.0115 + j0.2206
switching angles:	A = 71.8° (3.99 ms) B = 163.1° (9.06 ms) C = 32.0° (1.78 ms)

(note: Switching angles are used to simulate assynchronous switching.)

Line data

zero sequence:	R = 0.418 ohm/mi L = 5.198 mh/mi C = 0.01232 μf/mi
pos/neg sequence:	R = 0.0644 ohm/mi L = 1.629 mh/mi C = 0.01908 μf/mi
length:	126 mi

Load data open circuit

The results from this example are shown in the computer plot of the receiving end voltage in figure 3-1. The waveforms and amplitudes are in excellent agreement with "Surge" results. The "Surge" results have slightly lower

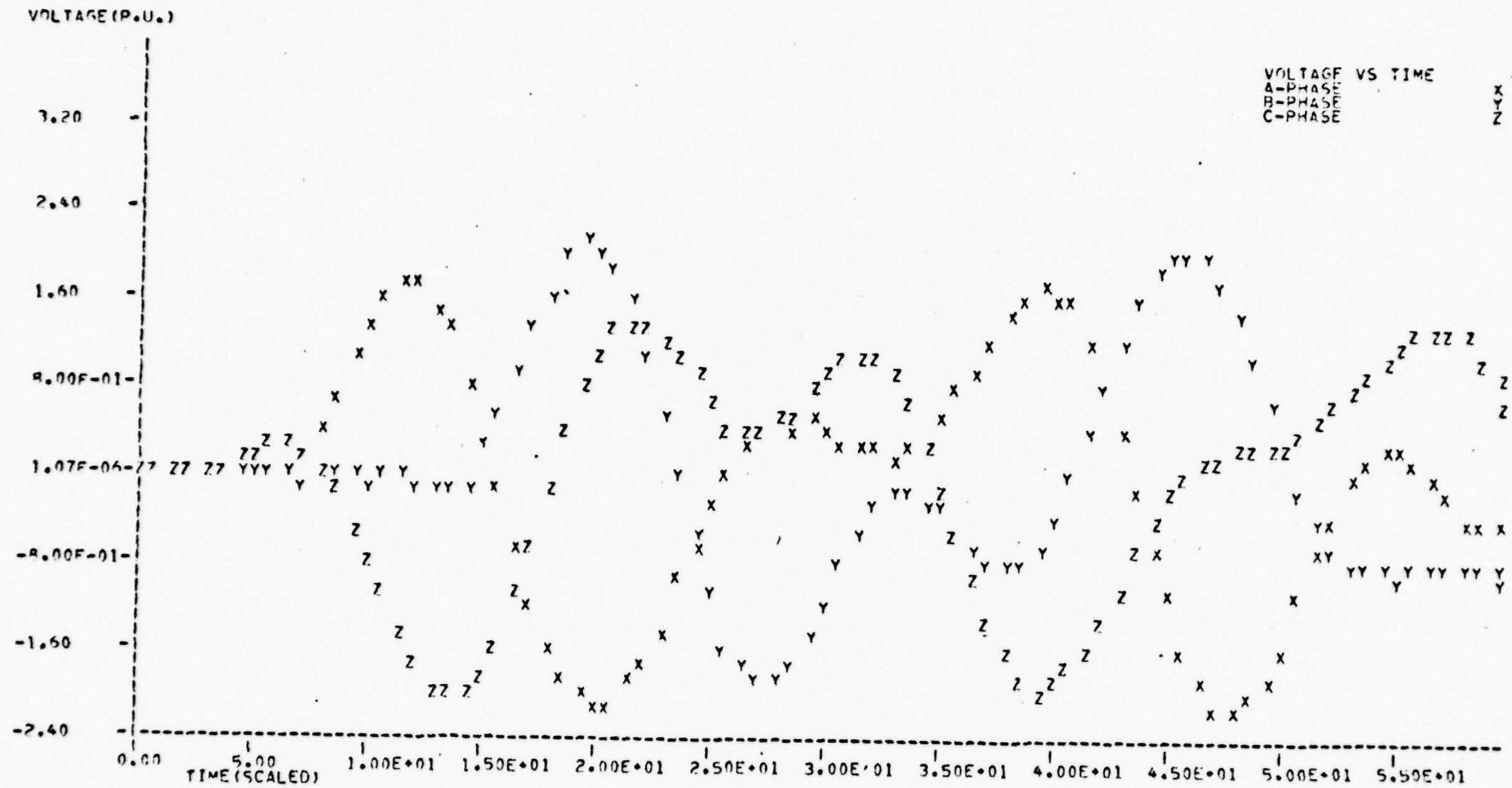

Figure 3-1. Three phase example of assynchronous switching

voltage maximum but that is to be expected since it handles the frequency-dependence of parameters which is discussed in the next section.

Frequency-Dependence of Line Parameters

This thesis has not attempted to include a method for handling frequency-dependent line parameters. Other works in this area have dealt with the problem, and since it does have a bearing on the transients observed on the line it will be mentioned here.

An overhead transmission line is composed of a certain number of phase conductors and neutrals. The phase conductors are separated from each other but the neutrals are connected through the towers and are thus grounded. The modeling of this ground return for inclusion into a model of the line is very complex due to the non-uniformity of the earth. In an early paper by Carson on this subject [14], he established the fact that for a single conductor with ground return its resistance and inductance per unit length are proportional to frequency (f in Hz). Another author [8] has noted Carson's results as:

$$R \propto (f)^k \tag{3-5a}$$

where,

$$0.5 \leq k \leq 1.0 \tag{3-5b}$$

and,

$$L \propto (f)^n \qquad (3\text{-}6a)$$

where,

$$-0.5 \leq n \leq 0 \qquad (3\text{-}6b)$$

In three phase transient analysis using modal techniques, the O-mode is often referred to as the ground mode. In a study of frequency effects on modal values by Hedman [5], it was found that the mode most affected by frequency is the ground mode. His conclusions on earth affects are listed below.

1. Carson's earth-correction terms produce the predominant earth-correction effects for a transmission line over an imperfect earth.

2. Carson's earth-correction resistance terms are proportional to frequency and to the square root of frequency, respectively in the low- and high-frequency regions.

3. Effects of the high relative-dielectric constant of the earth are significant only for frequencies higher than 0.5 MHz and when both earth resistivity and dielectric constant are high.

4. Earth correction for admittance terms appear to be unimportant for frequencies lower than 1 MHz.

5. Carson's earth correction terms significantly affect the modal voltages and eigenvectors for frequencies from 60 Hz to 1 MHz.

6. Modal analysis, using the perfect earth, should be adequate for radio-noise propagation studies.

7. For carrier-current analysis, earth effects become significant.

The frequency dependent resistance and inductance have a damping effect on transient voltages when compared to transients that do not consider frequency dependence.

In computer programs that deal with frequency dependence a frequency domain technique is used to determine the values of parameters in the equations already present. Methods such as the Fourier Transform [8, 15] and the Modified Fourier Transform [9], are used to evaluate the parameters over a range of frequencies at each time step in the program. A typical range of frequencies would be 0-12.8 kHz [8].

IV. CONCLUSION

The method for modeling the transmission line that results from the trapezoidal rule of integration is a very straight forward way of solving transmission line transients. In fact, at each time step the problem to be solved is that of a d.c. circuits problem. The argument for this is that the time step is selected small enough, i.e. the lossless travel time for the traveling wave to cross the section, that nothing changes during that span of time. Handling the end effects of the line using the trapezoidal rule proved to be very compatible with the rest of the line model.

The argument was made that as the number of sections increases while each section length decreases that this more closely approximated the actual line performance. This proved to be true in the sample lines of 2, 10, and 40 sections in the single phase case. The number of sections and the time step chosen were related by selecting a time step that would equal the lossless travel time for the section. A smaller time step proved to be more accurate for the smaller number of sectioned lines but about the same for the larger number of sectioned lines.

The modal technique used in the three phase case proved to be very powerful in handling three phase transients. Its decoupling of the phases into the modal values just presented the problem of solving three single phase cases. Transformation back to the phase values presented the desired results. Although frequency-dependence of line parameters was not included, it did not present a serious problem in the analysis. The analysis, as developed, produces slightly higher voltages than had frequency-dependence been included. Since the maximum voltages are of prime interest in transient analysis, this places this thesis' results on the conservative side in determining them.

Finally, the examples cited and run on the program developed present excellent agreement between this method and the methods previously developed.

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APPENDICES

APPENDIX A

INVERSION TECHNIQUE FOR A SPARSE MATRIX

INVERSION TECHNIQUE FOR A SPARSE MATRIX

The general form of the Y-matrix discussed in Chapter 2 is shown in figure A-1. In a typical computer routine for inverting a matrix, every entry would be used in determining the inverse. Since the Y-matrix is a tridiagonal matrix, it would be advantageous to exploit its sparseness for the computer.

The method chosen for inverting the matrix is the Gauss-Jordon method [12]. This method uses an augmented matrix composed of the matrix to be inverted and an identical sized identity matrix, as illustrated below.

$$[A \mid I] \tag{A-1a}$$

Row and/or column operations are performed on the matrix to be inverted while the same operations are performed on the identity matrix. When the A-matrix has been reduced to the identity matrix, the right side of the augmented matrix now contains A^{-1} as shown below.

$$[I \mid A^{-1}] \tag{A-1b}$$

Since the typical row entry of the Y-matrix only contains elements in the y_{ii} and $y_{i,i\pm1}$ positions, see

figure A-1, only these positions are dealt with in the computer routine. Also, in the computer routine developed only the y_{ii} entries are changed, while all operations that would normally be performed on the Y-matrix are done only on the identity matrix. The new diagonal entry is given by,

$$y'_{i+1,i+1} = y_{i+1,i+1} - (y_{i+1,i}/y'_{i,i})y_{i,i+1}, i=1,\ldots,n-1 \quad \text{(A-2)}$$

As these operations are performed, the Y-matrix is changed to upper-triangular form as shown below.

$$\begin{bmatrix} y_{11} & y_{12} & 0 & 0 & 0 \ldots 0 \\ 0 & y'_{22} & y_{23} & 0 & 0 \ldots 0 \\ 0 & 0 & y'_{33} & y_{34} & 0 \ldots 0 \end{bmatrix}$$

To further reduce Y to a diagonal matrix, row operations are performed to eliminate the off diagonal entries. Again operations are performed only on the identity matrix. Realizing that only the diagonal elements of the Y-matrix need to be changed, since all other elements are eliminated, saves the programmer and computer time. Now that the Y-matrix is in diagonal form, each row of the identity matrix is divided by the appropriate y'_{ii} element. The inverse of the Y-matrix is now formed in the place of the identity matrix and the Y-matrix is set equal to it.

$$
\begin{bmatrix}
y_{11} & y_{12} & 0 & 0 & 0 & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & 0 \\
y_{21} & y_{22} & y_{23} & 0 & 0 & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & 0 \\
0 & y_{32} & y_{33} & y_{34} & 0 & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & & & & & & & & \cdot \\
\cdot & & & \cdot & \cdot & \cdot & & & & & & & \cdot \\
\cdot & & & & \cdot & \cdot & \cdot & & & & & & \cdot \\
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\cdot & & & & & & & \cdot & \cdot & \cdot & & & \cdot \\
\cdot & & & & & & & & \cdot & \cdot & \cdot & & \cdot \\
0 & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & 0 & y_{n-2,n-3} & y_{n-2,n-2} & y_{n-2,n-1} & 0 \\
0 & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & 0 & 0 & y_{n-1,n-2} & y_{n-1,n-1} & y_{n-1,n} \\
0 & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & 0 & 0 & 0 & y_{n,n-1} & y_{nn}
\end{bmatrix}
$$

Figure A-1. General form of the Y-matrix

APPENDIX B

SCALING OF DATA FOR COMPUTER INPUT

SCALING OF DATA FOR COMPUTER INPUT

In order to avoid working with very small and very large numbers associated with a transmission line, a scaling method was devised to input data into the computer. Starting with the transmission line equations (primes denote per unit length),

$$\frac{\partial v_a}{\partial x_a} = (R'_a + L'_a \frac{\partial}{\partial t}) i \qquad \text{(B-1a)}$$

$$\frac{\partial i_a}{\partial x_a} = C'_a \frac{\partial v_a}{\partial t} \qquad \text{(B-2a)}$$

the bases for the individual values are chosen. The subscript a, denotes actual (SI) values. Let

$$x = \frac{x_a}{x_{base}} \qquad \text{(B-3a)}$$

with

$$x_{base} = \text{line length} = d \qquad \text{(B-3b)}$$

Let

$$L = \frac{L'_a}{L'_{base}} = 1 \qquad \text{(B-4a)}$$

where

$$L'_{base} = L'_a \qquad \text{(B-4b)}$$

Let

$$C = \frac{C'_a}{C'_{base}} = 1 \qquad \text{(B-5a)}$$

where

$$C'_{base} = C'_a \qquad \text{(B-5b)}$$

In three phase analysis, L'_{base} and C'_{base} are chosen to be the positive sequence L'_1 and C'_1. Time is scaled as

$$t = \frac{t_a}{t_{base}} \qquad \text{(B-6a)}$$

where

t_{base} = lossless travel time for the line

$$t_{base} = \sqrt{L'_a C'_a}\, d \qquad \text{(B-6b)}$$

Again, in three phase analysis, L'_{base} and C'_{base} are chosen to be the positive sequence L'_1 and C'_1. The voltage is scaled as

$$v = \frac{v_a}{v_{base}} \qquad \text{(B-7a)}$$

where

$v_{base} = v_{LN}$ (line to neutral rated maximum) (B-7b)

To more clearly illustrate v_{base}, assume a 500 KV system. Then,

$$v_{base} = \frac{500\sqrt{2}}{\sqrt{3}} \text{ KV}$$

Let

z_{base} = lossless characteristic impedance

where

$$z_{base} = \sqrt{\frac{L'_a}{C'_a}} \qquad \text{(B-8)}$$

Let

$$i_{base} = v_{base}/z_{base} \qquad \text{(B-9)}$$

Substituting these values into equation B-1a and B-2a,

$$\frac{\partial(vv_{base})}{\partial(xd)} = (R'_a + LL'_{base} \frac{\partial}{\partial(td\sqrt{L'_aC'_a})})ii_{base} \qquad \text{(B-1b)}$$

and,

$$\frac{\partial(ii_{base})}{\partial(xd)} = CC_{base} \frac{\partial(vv_{base})}{\partial(td\sqrt{L'_aC'_a})} \qquad \text{(B-2b)}$$

Clearing terms on each side of equations B-1b and B-2b,

$$\frac{\partial v}{\partial x} = (\frac{R'_a}{z_{base}} + L \frac{\partial}{\partial t})i \qquad \text{(B-1c)}$$

and,

$$\frac{\partial i}{\partial x} = C \frac{\partial v}{\partial t} \qquad \text{(B-2c)}$$

Letting,

$$R = \frac{R'_a}{z_{base}} \qquad \text{(B-1d)}$$

equation B-1c becomes,

$$\frac{\partial v}{\partial x} = (R + L \frac{\partial}{\partial t})i \qquad \text{(B-1e)}$$

The only parameter now left to scale is frequency. Frequency is scaled by keeping

$$\omega t = \omega_a t_a \tag{B-10a}$$

But time has already been scaled and

$$\frac{\omega_a t_a}{\omega_{base} t_{base}} = \omega t \tag{B-10b}$$

$$\therefore \quad \omega_{base} = \frac{1}{t_{base}} = \frac{1}{\sqrt{L'_a C'_a}\, d} \tag{B-10c}$$

For scaling inductance (ℓ) and capacitance (c) that are not per unit length, the following method is used.

$$\frac{R_a + j\omega_a \ell_a}{z_{base}} = R + j\omega\ell \tag{B-11a}$$

$$\frac{\omega_a \ell_a}{\sqrt{\frac{L'_a}{C'_a}}} = \frac{\omega_a}{\left(\frac{1}{\sqrt{L'_a C'_a}\, d}\right)} \ell \tag{B-11b}$$

$$\ell_a = (L'_a d)\ell \tag{B-11c}$$

$$\therefore \quad \ell_{base} = L'_a d \tag{B-11d}$$

Similarly,

$$c_{base} = C'_a d \tag{B-12}$$

An example will help to clarify the scaling method. Using the three phase example of Chapter 3, all values will be scaled as follows:

Base data

$$x_{base} = 126 \text{ mi} = d$$

$$L'_{base} = 1.629 \text{ mh/mi}$$

$$C'_{base} = 0.01908 \ \mu\text{f/mi}$$

$$t_{base} = \sqrt{L'_1 C'_1}\, d = .7024 \text{ ms}$$

$$v_{base} = \frac{345\sqrt{2}}{\sqrt{3}} = 282 \text{ KV}$$

$$z_{base} = \sqrt{\frac{L'_1}{C'_1}} = 292 \text{ ohms}$$

$$\omega_{base} = 1/t_{base} = 1423.69 \text{ rad/s}$$

$$\ell_{base} = L'_1 d = 0.2053 \text{ H}$$

$$c_{base} = C'_1 d = 2.4041 \ \mu\text{f}$$

Source data

$$Em = 1.0011$$

$$\omega = [2\pi(50)]/1423.69 = 0.2207$$

$$\theta = 0°$$

$$R = [0.0115(345)^2/100]/292 = 0.0469$$

$$L = [(0.2206(345)^2/100)2\ (50)]/0.2053 = 4.0710$$

(note: switching angles are converted to times)

$$T_A = (71.8°/360°)(1/50)/.7024(10^{-3}) = 5.6789$$

$$T_B = (163.1°/360°)(1/50)/.7024(10^{-3}) = 12.9002$$

$$T_C = (32°/360°)(1/50)/.7024(10^{-3}) = 2.5310$$

Line data

zero sequence: $R = 0.418(126)/292 = 0.1804$

$L = 5.198(10^{-3})/1.629(10^{-3}) = 3.1909$

$C = 0.01232(10^{-6})/0.01908(10^{-6}) = 0.6457$

pos/neg sequence: $R = 0.0644(126)/292 = 0.0278$

$L = 1.629(10^{-3})/1.629(10^{-3}) = 1.0$

$C = 0.01908(10^{-6})/0.01908(10^{-6}) = 1.0$

Load data

open circuit

The bases for scaling are summarized in Table B-1.

Table B-1. Summary of base values for scaling

Parameter	Base	
	1ϕ	3ϕ
x	d	d
z	$\sqrt{L'_a/C'_a}$	$\sqrt{L'_1/C'_1}$
R	$\sqrt{L'_a/C'_a}$	$\sqrt{L'_1/C'_1}$
R'	$\sqrt{L'_a/C'_a}/d$	$\sqrt{L'_1/C'_1}/d$
L'	L'_a	L'_1
C'	C'_a	C'_1
t	$\sqrt{L'_aC'_a}d$	$\sqrt{L'_1C'_1}d$
ω	$1/\sqrt{L'_aC'_a}d$	$1/\sqrt{L'_1C'_1}d$
ℓ	L'_ad	L'_1d
c	C'_ad	C'_1d
V	V_{LN}(peak)	V_{LN}(peak)
I	$V_{LN}/\sqrt{L'_a/C'_a}$	$V_{LN}/\sqrt{L'_1/C'_1}$

APPENDIX C

FORTRAN COMPUTER PROGRAM

FORTRAN COMPUTER PROGRAM

User's Guide

In order to get data into the single and three phase computer programs, TTL, it must be scaled as follows. (Note: primes denote per unit length; a = actual value)

$$V_{base} = V_{LN} \text{ (system peak)}$$

$$L'_{base} = L'_a \text{ (}L'_1 \text{ for } 3\phi\text{)}$$

$$C'_{base} = C'_a \text{ (}C'_1 \text{ for } 3\phi\text{)}$$

$$X_{base} = d \text{ (line length)}$$

$$Z_{base} = \sqrt{L'_a/C'_a}$$

$$R'_{base} = Z_{base}/d$$

$$\ell_{base} = L'_a d$$

$$c_{base} = C'_a d$$

$$I_{base} = V_{base}/Z_{base}$$

$$t_{base} = \sqrt{L'_a C'_a}\ d$$

$$\omega_{base} = 1/\sqrt{L'_a C'_a}\ d$$

To obtain scaled data, divide each individual parameter by its base. The data cards are as follows:

```
1-   TMAX   , TPLOT  ,DTPLOT , LX   , NP   ,IPLOT       (1φ & 3φ)
   (F10.4) (F10.4) (F10.4) (I10) (I10) (I10)
```

TMAX - maximum scaled problem time for program to run; at least 2 cycles, scaled, for a sinusoidal source

TPLOT - scaled problem time at which voltage versus position is plotted. Cannot be zero.

DTPLOT - scaled problem time periods after TPLOT at which subsequent plots are made. Cannot be zero.

LX - node at which voltage vs time is plotted; $1 \leq LX \leq N=1$.

NP - determines number of points plotted in voltage versus time, i.e., every NP points. The calculating Δt is fixed internally at 1/N (see Section 2). The plotting, and printing, time increment is $NP*\Delta t$.

IPLOT - plot option

1. voltage versus position
2. voltage versus time
3. both

```
2-     R    ,   L    ,   C    ,  N                     (1φ)
   (F10.4) (F10.4) (F10.4) (I10)
```

```
2-    RA    ,   LA   ,   CA   ,  RB    ,  LB    ,  CB    ,   (3φ)
   (F10.4) (F10.4) (F10.4) (F10.4) (F10.4) (F10.4)

     N
   (I10)
```

R - scaled line resistance; RA and RB are scaled zero and positive sequence values respectively

L - scaled line inductance; LA and LB are scaled sequence values

C - scaled line capacitance; CA and CB are scaled sequence values

N - number of line sections; the line length divided by 15 km (9.3 mi) to the nearest whole number, maximum number is 48

```
3-   EMAX  , OMEGA , THETA ,  RS   ,  LS                    (1ϕ)
   (F10.4) (F10.4) (F10.4) (F10.4) (F10.4)

     EMAX  , OMEGA ,  RSA  ,  LSA  ,  RSB  ,  RSC           (3ϕ)
   (F10.4) (F10.4) (F10.4) (F10.4) (F10.4) (F10.4)
```

EMAX - maximum, peak value (line to neutral for 3ϕ) of the voltage source, usually 1.0

OMEGA - $2\pi f$

THETA - phase shift in radians

RS - scaled source resistance; RSA and RSB are scaled zero and positive sequence values respectively

LS - scaled source inductance; LSA and LSA are scaled sequence values

```
4-    TA   ,  TB   ,  TC   , THETA                          (3ϕ)
   (F10.4) (F10.4) (F10.4) (F10.4)
```

TA - scaled time delay for a-phase

TB - scaled time delay for b-phase

TC - scaled time delay for c-phase

THETA - phase shift in radians

```
4-    GLL  , GAML  ,  CL                                    (1ϕ)
   (F10.4) (F10.4) (F10.4)

5-   GLLA  , GAMLA ,  CLA  , GLLB  , GAMLB ,  CLB           (3ϕ)
   (F10.4) (F10.4) (F10.4) (F10.4) (F10.4) (F10.4)
```

GLL - scaled load conductance; GLLA and GLLB are scaled zero and positive sequence values respectively

GAML - scaled load gamma (1/inductance); GAMLA and GAMLB are scaled sequence values

CL - scaled load capacitance; CLA and CLB are scaled sequence values

(Note: the load can be any parallel combination of inductance, resistance, and capacitance.)

If a value is left blank on a data card it will be interpreted as zero in the computer. For programs that run for long periods, the JCL cards controlling run time may have to be changed. Always check the last data cards to insure that they correspond to the plot options chosen, since they label the plots.

Program Listings

```
/JOB JOB,PASSES=40,TIME=200
      REAL L,IS,IB,IL,ILL,ICL,ILS,ICS,LS,IG
      DIMENSION XJ(50),XLAB(5),YLAB(5),GLAB(5),DATLAB(5)
      DIMENSION Y(50,50),V(50),IB(50),PV(50),PIB(50),CM(50),B(50,50),CC
     1(50),CV(50),TIME(900),VOLT(900),AMPS(900)
      READ(5,320)TMAX,TPLOT,DTPLOT,LX,NP,IPLOT
      READ(5,330)R,L,C,N
      READ(5,340)EMAX,OMEGA,THETA,RS,LS
      READ(5,350)GLL,CL,GAML
      WRITE(6,360)
      WRITE(6,370)
      WRITE(6,380)
      WRITE(6,390)TMAX,TPLOT,DTPLOT,LX,NP,IPLOT
      WRITE(6,400)
      WRITE(6,410)
      WRITE(6,420)R,L,C,N
      WRITE(6,430)
      WRITE(6,440)
      WRITE(6,450)EMAX,OMEGA,THETA,RS,LS
      WRITE(6,460)
      WRITE(6,470)
      WRITE(6,480)GAML,CL,GLL
      WRITE(6,490)
      NC=NP
      T=0.
      DX=1./N
      N=N+1
      DT=DX
      L=L*DX
      C=C*DX
      R=R*DX
      LS=LS+.0000001
      NN=N+1
      NPTS=(TMAX*(N-1))/NP+1
      DO 10 I=1,NN
      IB(I)=0.
      V(I)=0.
      PV(I)=0.
      PIB(I)=0.
      CM(I)=0.
      CC(I)=0.
      CV(I)=0.
      DO 10 J=1,NN
      Y(I,J)=0.
   10 B(I,J)=0.
      GG=1000000.
      GGS=1./((2.*LS/DT)+RS)
      GL=DT*GAML/2.+(2.*CL/DT)+GLL
      GS=1./((2.*L/DT)+R)
      GP=2.*C/DT
C****BUILD THE Y-MATRIX****
      Y(1,1)=GG+GGS
      Y(1,2)=-GGS
      Y(2,1)=-GGS
      DO 50 I=2,NN
      DO 50 J=2,NN
      IF(J.NE.I)GO TO 30
      IF(I.NE.2)GO TO 20
      Y(I,J)=Y(I,J)+GGS+GS
```

```
      GO TO 50
   20 IF(I.NE.NN)GO TO 40
      Y(I,J)=Y(I,J)+GS+GP+GL
      GO TO 50
   30 IF(J.NE.I+1.AND.J.NE.I-1)GO TO 50
      Y(I,J)=Y(I,J)-GS
      GO TO 50
   40 Y(I,J)=Y(I,J)+2.*GS+GP
   50 CONTINUE
C****INVERT Y-MATRIX****
      DO 60 I=1,NN
   60 B(I,I)=1.
      DO 70 I=1,N
      RATIO=-Y(I+1,I)/Y(I,I)
      Y(I+1,I+1)=Y(I+1,I+1)+RATIO*Y(I,I+1)
      DO 70 J=1,NN
      IF(ABS(RATIO*B(I,J)).LT.1E-10)GO TO 70
      B(I+1,J)=B(I+1,J)+RATIO*B(I,J)
   70 CONTINUE
      DO 80 I=1,N
      K=NN-I
      DO 80 J=1,NN
      RATIO=-Y(K,K+1)/Y(K+1,K+1)
      IF(ABS(RATIO*B(K+1,J)).LT.1E-10)GO TO 80
      B(K,J)=B(K,J)+RATIO*B(K+1,J)
   80 CONTINUE
      DO 90 I=1,NN
      DO 90 J=1,NN
   90 B(I,J)=B(I,J)/Y(I,I)
      DO 110 I=1,NN
      DO 110 J=1,NN
      IF(ABS(B(I,J)).LT.1E-10)GO TO 100
      Y(I,J)=B(I,J)
      GO TO 110
  100 Y(I,J)=0.0
  110 CONTINUE
      VMAX=0.
      PILL=0.
      PICL=0.
      JX=1
      IF(IPLOT.EQ.0)GO TO 130
      IF(IPLOT-2)120,130,120
  120 READ(5,310)XLAB,YLAB,GLAB,DATLAB
  130 CONTINUE
      IF(T.GT.TMAX)GO TO 260
      CC(1)=GGS*(PV(1)-PV(2))+PIB(1)*((2.*LS/DT)-RS)/((2.*LS/DT)+RS)
      DO 140 K=2,N
  140 CC(K)=GS*(PV(K)-PV(K+1))+PIB(K)*((2.*L/DT)-R)/((2.*L/DT)+R)
      DO 150 J=3,NN
  150 CV(J)=PIB(J-1)-PIB(J)+GP*PV(J)
      CLL=(DT*GAML/2.)*PV(NN)+PILL
      CCL=-(2.*CL/DT)*PV(NN)-PICL
      IL=CLL+CCL
      ANG=OMEGA*T+THETA
      IS=FMAX*COS(ANG)
      IG=IS*1000000.
      CM(1)=IG-CC(1)
      DO 160 J=2,N
  160 CM(J)=CG(J-1)+CV(J)-CC(J)
```

```
      CM(NN)=CC(N)+CV(NN)-IL
      DO 170 J=1,NN
      V(J)=0.0
      DO 170 K=1,NN
      IF(ABS(Y(J,K)*CM(K)).LT.1E-10)GO TO 170
      V(J)=V(J)+Y(J,K)*CM(K)
  170 CONTINUE
      IB(1)=GGS*(V(1)-V(2))+CC(1)
      DO 180 J=2,N
  180 IB(J)=GS*(V(J)-V(J+1))+CC(J)
      IB(NN)=GL*V(NN)+IL
      IF(ABS(PV(LX+1)).LT.ABS(VMAX))GO TO 190
      VMAX=PV(LX+1)
  190 CONTINUE
      ILL=(DT*GAML/2.)*V(NN)+CLL
      ICL=(2.*CL/DT)*V(NN)+CCL
      IF(NP-NC)200,200,240
  200 IF(T.LT.TPLOT)GO TO 230
      WRITE(6,550)T
      WRITE(6,540)
      DO 210 J=2,NN
      JJ=J-1
      XJ(JJ)=JJ
      PV(JJ)=PV(J)
      PIB(JJ)=PIB(J)
  210 WRITE(6,560)JJ,PV(JJ),PIB(JJ)
      WRITE(6,570)
      IF(IPLOT.EQ.0)GO TO 230
      IF(IPLOT-2)220,230,220
  220 CALL GRAPH(N,XJ,PV,11,7,10.0,8.0,0.0,1.0,0.0,-5.0,XLAB,YLAB,
     1GLAB,DATLAB)
      TPLOT=TPLOT+DTPLOT
  230 CONTINUE
      NC=0.0
      VOLT(JX)=PV(LX)
      AMPS(JX)=PIB(LX)
      TIME(JX)=T
      JX=JX+1
  240 NC=NC+1
      DO 250 J=1,NN
      PV(J)=V(J)
  250 PIB(J)=IB(J)
      PILL=ILL
      PICL=ICL
      T=T+DT
      GO TO 130
  260 CONTINUE
      IF(IPLOT.EQ.0)GO TO 300
      IF(IPLOT-2)300,270,270
  270 WRITE(6,500)VMAX
      WRITE(6,510)LX
      WRITE(6,520)
      DO 280 I=1,NPTS
  280 WRITE(6,530)TIME(I),VOLT(I),AMPS(I)
  290 READ(5,310)XLAB,YLAB,GLAB,DATLAB
      CALL GRAPH(NPTS,TIME,VOLT,11,7,12.0,8.0,0.0,0.0,0.0,-5.0,XLAB,
     1YLAB,GLAB,DATLAB)
  300 CONTINUE
  310 FORMAT(20A4)
```

```
  320 FORMAT(3F10.4,3I10)
  330 FORMAT(3F10.4,2I10)
  340 FORMAT(5F10.4)
  350 FORMAT(3F10.4)
  360 FORMAT('1','**************************************************INPU
     1T DATA*******************************************',//)
  370 FORMAT(44X,'****OUTPUT CONTROL****',//)
  380 FORMAT(27X,'TMAX',6X,'TPLOT',4X,'DTPLOT',8X,'LX',8X,'NP',6X,'IPLOT
     1',//)
  390 FORMAT(22X,3F10.3,3I10)
  400 FORMAT(//,43X,'****SCALED LINE DATA****',//)
  410 FORMAT(29X,'RESISTANCE',2X,'INDUCTANCE',2X,'CAPACITANCE',6X,
     1'SECTIONS',//)
  420 FORMAT(27X,F10.3,2X,F10.3,3X,F10.3,2X,I10)
  430 FORMAT(//,41X,'****SCALED SOURCE DATA****',//)
  440 FORMAT(31X,'EMAX',5X,'OMEGA',5X,'THETA',2X,'RESISTANCE',2X,
     1'INDUCTANCE',//)
  450 FORMAT(25X,5F10.3)
  460 FORMAT(//,43X,'****SCALED LOAD DATA****',//)
  470 FORMAT(39X,'GAMMA',2X,'CAPACITANCE',2X,'CONDUCTANCE',//)
  480 FORMAT(34X,F10.3,1X,F10.3,3X,F10.3)
  490 FORMAT(///,'*************************************************OUTPU
     1T DATA**********************************************',//)
  500 FORMAT(5X,'VMAX=',F10.3,//)
  510 FORMAT(5X,'NODE=',I2,/)
  520 FORMAT(/,5X,'TIME',4X,'VOLTAGE',3X,'CURRENT',/)
  530 FORMAT(10F10.4)
  540 FORMAT(/,6X,'NODE',3X,'VOLTAGE',3X,'CURRENT',/)
  550 FORMAT(///,5X,'TIME =',F6.3)
  560 FORMAT(I10,8F10.4)
  570 FORMAT(/,'**************************************************',/)
      STOP
      END

/GO

/DATA

/DATA
```

```
/JOB GROSS,PAGES=100,TIME=300
      REALIA,IB,IC,ISA,ISB,ISC,IGA,IGB,IGC,LA,LB,LC,IMAX,IBA,IBB,IBC,
     1ILA,ILB,ILC,ILLA,ILLB,ILLC,ICLA,ICLB,ICLC,ILSA,ILSB,ILSC,ICSA,
     2ICSB,ICSC,LSA,LSB,LSC
      DIMENSION YA(50,50),YB(50,50),YC(50,50),VA(50),VB(50),VC(50),
     1IBA(50),IBB(50),IBC(50),PVA(50),PVB(50),PVC(50),PIBA(50),PIBB(50),
     2PIBC(50),CMA(50),CMB(50),CMC(50),CCA(50),CCB(50),CCC(50),CVA(50),
     3CVB(50),CVC(50),TIME(500),AVOLT(500),BVOLT(500),CVOLT(500),AAMPS
     4(500),BAMPS(500),CAMPS(500),XJ(50),XLAB(5),YLAB(5),GLAB(5),
     5DATLAB(5),DATLAA(5),DATLAC(5)
      READ(5,300)TMAX,TPLOT,DTPLOT,LX,NP,IPLOT
      READ(5,310)RA,LA,CA,RB,LB,CB,N
      READ(5,320)EMAX,OMEGA,RSA,LSA,RSB,LSB
      READ(5,320)TA,TB,TC,THETA
      READ(5,330)GLLA,GAMLA,CLA,GLLB,GAMLB,CLB
      WRITE(6,340)
      WRITE(6,350)
      WRITE(6,360)
      WRITE(6,370)TMAX,TPLOT,DTPLOT,LX,NP,IPLOT
      WRITE(6,380)
      WRITE(6,390)N
      WRITE(6,400)
      WRITE(6,410)
      WRITE(6,420)RA,LA,CA
      WRITE(6,430)
      WRITE(6,440)RB,LB,CB
      WRITE(6,450)
      WRITE(6,460)
      WRITE(6,470)EMAX,OMEGA,TA,TB,TC,THETA
      WRITE(6,480)
      WRITE(6,490)
      WRITE(6,500)RSA,LSA
      WRITE(6,510)
      WRITE(6,520)RSB,LSB
      WRITE(6,530)
      WRITE(6,540)
      WRITE(6,550)
      WRITE(6,560)GLLA,GAMLA,CLA
      WRITE(6,570)
      WRITE(6,580)GLLB,GAMLB,CLB
      WRITE(6,590)
      DX=1./N
      N=N+1
      NN=N+1
      NPTS=(TMAX*(N-1))/NP+1
      DT=DX
      NC=NP
      T=0.
      LA=LA*DX
      LB=LB*DX
      LC=LB
      CA=CA*DX
      CB=CB*DX
      CC=C
      RA=RA*DX
      RB=RB*DX
      RC=RB
      RSC=RSB
      LSA=LSA+.0000001
```

```
      LSB=LSB+.0000001
      LSC=LSB
      GAMLC=GAMLB
      CLC=CLB
      GGA=1000000.
      GGB=GGA
      GGC=GGA
      GGSA=1./((2.*LSA/DT)+RSA)
      GLA=(DT*GAMLA/2.)+(2.*CLA/DT)+GLLA
      GSA=1./((2.*LA/DT)+RA)
      GPA=2.*CA/DT
C******SUBROUTINE TO BUILD AND INVERT THE Y-MATRIX
      CALL YINVRT(YA,GGA,GLA,GSA,GPA,NN,GGSA)
      GGSB=1./((2.*LSB/DT)+RSB)
      GLB=(DT*GAMLB/2.)+(2.*CLB/DT)+GLLB
      GSB=1./((2.*LB/DT)+RB)
      GPB=2.*CB/DT
      GGSC=GGSB
      GLC=GLB
      GSC=GSB
      GPC=GPB
      CALL YINVRT(YB,GGB,GLB,GSB,GPB,NN,GGSB)
      DO 10 I=1,NN
      IBA(I)=0.
      IBB(I)=0.
      IBC(I)=0.
      VA(I)=0.
      VB(I)=0.
      VC(I)=0.
      PVA(I)=0.
      PVB(I)=0.
      PVC(I)=0.
      PIBA(I)=0.
      PIBB(I)=0.
      PIBC(I)=0.
      CMA(I)=0.
      CMB(I)=0.
      CMC(I)=0.
      CCA(I)=0.
      CCB(I)=0.
      CCC(I)=0.
      CVA(I)=0.
      CVB(I)=0.
      CVC(I)=0.
      DO 10 J=1,NN
   10 YC(I,J)=YB(I,J)
      PILLA=0.
      PILLB=0.
      PILLC=0.
      PICLA=0.
      PICLB=0.
      PICLC=0.
      JX=1
      VAMAX=0.
      VBMAX=0.
      VCMAX=0.
      IF(IPLOT.EQ.0)GO TO 30
      IF(IPLOT-2)20,30,20
   20 READ(5,640)XLAB,YLAB,GLAB,DATLAA
```

```
      READ(5,640)DATLAB
      READ(5,640)DATLAC
   30 CONTINUE
      IF(T.GT.TMAX)GO TO 260
      CCA(1)=GGSA*(PVA(1)-PVA(2))+PIBA(1)*((2.*LSA/DT)-RSA)/
     1((2.*LSA/DT)+RSA)
      CCB(1)=GGSB*(PVB(1)-PVB(2))+PIBB(1)*((2.*LSB/DT)-RSB)/
     1((2.*LSB/DT)+RSB)
      CCC(1)=GGSC*(PVC(1)-PVC(2))+PIBC(1)*((2.*LSC/DT)-RSC)/
     1((2.*LSC/DT)+RSC)
      DO 40 K=2,N
      CCA(K)=GSA*(PVA(K)-PVA(K+1))+PIBA(K)*((2.*LA/DT)-RA)/
     1((2.*LA/DT)+RA)
      CCB(K)=GSB*(PVB(K)-PVB(K+1))+PIBB(K)*((2.*LB/DT)-RB)/
     1((2.*LB/DT)+RB)
   40 CCC(K)=GSC*(PVC(K)-PVC(K+1))+PIBC(K)*((2.*LC/DT)-RC)/
     1((2.*LC/DT)+RC)
      DO 50 J=3,NN
      CVA(J)=PIBA(J-1)-PIBA(J)+GPA*PVA(J)
      CVB(J)=PIBB(J-1)-PIBB(J)+GPB*PVB(J)
   50 CVC(J)=PIBC(J-1)-PIBC(J)+GPC*PVC(J)
      CLLA=(DT*GAMLA/2.)*PVA(NN)+PILLA
      CLLB=(DT*GAMLB/2.)*PVB(NN)+PILLB
      CLLC=(DT*GAMLC/2.)*PVC(NN)+PILLC
      CCLA=-(2.*CLA/DT)*PVA(NN)-PICLA
      CCLB=-(2.*CLB/DT)*PVB(NN)-PICLB
      CCLC=-(2.*CLC/DT)*PVC(NN)-PICLC
      ILA=CLLA+CCLA
      ILB=CLLB+CCLB
      ILC=CLLC+CCLC
      IF(T-TA)60,70,70
   60 IA=0.
      GO TO 80
   70 ANGA=OMEGA*T+THETA
      IA=EMAX*COS(ANGA)
   80 IF(T-TB)90,100,100
   90 IB=0.
      GO TO 110
  100 ANGB=OMEGA*T-2.0943951+THETA
      IB=EMAX*COS(ANGB)
  110 IF(T-TC)120,130,130
  120 IC=0.
      GO TO 140
  130 ANGC=OMEGA*T+2.0943951+THETA
      IC=EMAX*COS(ANGC)
  140 CONTINUE
      ISA=(IA+IB+IC)/3.
      ISB=(IA-IB)/3.
      ISC=(IA-IC)/3.
      IGA=ISA*1000000.
      IGB=ISB*1000000.
      IGC=ISC*1000000.
      CMA(1)=IGA-CCA(1)
      CMB(1)=IGB-CCB(1)
      CMC(1)=IGC-CCC(1)
      DO 150 J=2,N
      CMA(J)=CCA(J-1)+CVA(J)-CCA(J)
      CMB(J)=CCB(J-1)+CVB(J)-CCB(J)
  150 CMC(J)=CCC(J-1)+CVC(J)-CCC(J)
```

```
      CMA(NN)=CCA(N)+CVA(NN)-ILA
      CMB(NN)=CCB(N)+CVB(NN)-ILB
      CMC(NN)=CCC(N)+CVC(NN)-ILC
C******SUBROUTINE TO CALCULATE NODE VOLTAGES
      CALL VXCM(VA,YA,CMA,NN)
      CALL VXCM(VB,YB,CMB,NN)
      CALL VXCM(VC,YC,CMC,NN)
      IBA(1)=GGSA*(VA(1)-VA(2))+CCA(1)
      IBB(1)=GGSB*(VB(1)-VB(2))+CCB(1)
      IBC(1)=GGSC*(VC(1)-VC(2))+CCC(1)
      DO 160 K=2,N
      IBA(K)=GSA*(VA(K)-VA(K+1))+CCA(K)
      IBB(K)=GSB*(VB(K)-VB(K+1))+CCB(K)
  160 IBC(K)=GSC*(VC(K)-VC(K+1))+CCC(K)
      IBA(NN)=GLA*VA(NN)+ILA
      IBB(NN)=GLB*VB(NN)+ILB
      IBC(NN)=GLC*VC(NN)+ILC
      ILLA=(DT*GAMLA/2.)*VA(NN)+CLLA
      ILLB=(DT*GAMLB/2.)*VB(NN)+CLLB
      ILLC=(DT*GAMLC/2.)*VC(NN)+CLLC
      ICLA=(2.*CLA/DT)*VA(NN)+CCLA
      ICLB=(2.*CLB/DT)*VB(NN)+CCLB
      ICLC=(2.*CLC/DT)*VC(NN)+CCLC
C*******SUBROUTINE TO CALCULATE PHASE VALUES
      CALL MOTOPH(PVA,PVB,PVC,PIBA,PIBB,PIBC,NN)
      IF(ABS(PVA(LX+1)).LT.ABS(VAMAX))GO TO 170
      VAMAX=PVA(LX+1)
  170 IF(ABS(PVB(LX+1)).LT.ABS(VBMAX))GO TO 180
      VBMAX=PVB(LX+1)
  180 IF(ABS(PVC(LX+1)).LT.ABS(VCMAX))GO TO 190
      VCMAX=PVC(LX+1)
  190 CONTINUE
      IF(NP-NC)200,200,240
  200 IF(T.LT.TPLOT)GO TO 230
      WRITE(6,600)T
      WRITE(6,610)
      DO 210 J=2,NN
      JJ=J-1
      XJ(JJ)=JJ
      PVA(JJ)=PVA(J)
      PVB(JJ)=PVB(J)
      PVC(JJ)=PVC(J)
      PIBA(JJ)=PIBA(J)
      PIBB(JJ)=PIBB(J)
      PIBC(JJ)=PIBC(J)
  210 WRITE(6,620)JJ,PVA(JJ),PVB(JJ),PVC(JJ),PIBA(JJ),PIBB(JJ),PIBC(JJ)
      WRITE(6,630)
      IF(IPLOT.EQ.0)GO TO 230
      IF(IPLOT-2)220,230,220
  220 CALL GRAPH(NN,XJ,PVA,11,7,10.0,8.0,0.0,1.0,0.0,-4.0,XLAB,YLAB,
     1GLAB,DATLAA)
      CALL GRAPH(NN,XJ,PVB,11,7,10.0,8.0,0.0,1.0,0.0,-4.0,XLAB,YLAB,
     1GLAB,DATLAB)
      CALL GRAPH(NN,XJ,PVC,11,7,10.0,8.0,0.0,1.0,0.0,-4.0,XLAB,YLAB,
     1GLAB,DATLAC)
      TPLOT=TPLOT+DTPLOT
  230 CONTINUE
      NC=0
      AVOLT(JX)=PVA(LX)
```

```
      BVOLT(JX)=PVB(LX)
      CVOLT(JX)=PVC(LX)
      AAMPS(JX)=PIBA(LX)
      BAMPS(JX)=PIBB(LX)
      CAMPS(JX)=PIBC(LX)
      TIME(JX)=T
      JX=JX+1
  240 NC=NC+1
      DO 250 J=1,NN
      PVA(J)=VA(J)
      PVB(J)=VB(J)
      PVC(J)=VC(J)
      PIBA(J)=IBA(J)
      PIBB(J)=IBB(J)
  250 PIBC(J)=IBC(J)
      PILLA=ILLA
      PILLB=ILLB
      PILLC=ILLC
      PICLA=ICLA
      PICLB=ICLB
      PICLC=ICLC
      T=T+DT
      GO TO 30
  260 CONTINUE
      IF(IPLOT.EQ.0)GO TO 290
      IF(IPLOT-2)290,270,270
  270 WRITE(6,680)
      WRITE(6,690)
      WRITE(6,700)VAMAX,VBMAX,VCMAX
      WRITE(6,650)LX
      WRITE(6,660)
      DO 280 I=1,NPTS
  280 WRITE(6,670)TIME(I),AVOLT(I),BVOLT(I),CVOLT(I),AAMPS(I),BAMPS(I),C
     1AMPS(I)
      READ(5,640)XLAB,YLAB,GLAB,DATLAB
      CALL GRAPH(NPTS,TIME,AVOLT,4,107,12.0,8.0,0.0,0.0,0.0,-5.0,XLAB,
     1YLAB,GLAB,DATLAB)
      READ(5,640)DATLAB
      CALL GRAPH(NPTS,TIME,BVOLT,9,107,0.0,8.0,0.0,0.0,0.0,-5.0,XLAB,
     1YLAB,GLAB,DATLAB)
      READ(5,640)DATLAB
      CALL GRAPH(NPTS,TIME,CVOLT,8,107,0.0,8.0,0.0,0.0,0.0,-5.0,XLAB,
     1YLAB,GLAB,DATLAB)
  290 CONTINUE
  300 FORMAT(3F10.4,3I10)
  310 FORMAT(6F10.4,2I10)
  320 FORMAT(8F10.4)
  330 FORMAT(6F10.4)
  340 FORMAT('1','**************************************************INPU
     1T DATA**************************************************',//)
  350 FORMAT(44X,'****OUTPUT CONTROL****',//)
  360 FORMAT(27X,'TMAX',6X,'TPLOT',4X,'DTPLOT',8X,'LX',8X,'NP',6X,'IPLOT
     1',//)
  370 FORMAT(22X,3F10.3,3I10)
  380 FORMAT(//,43X,'****SCALED LINE DATA****',//)
  390 FORMAT(49X,'SECTIONS=',I2,//)
  400 FORMAT(37X,'RESISTANCE',2X,'INDUCTANCE',2X,'CAPACITANCE',//)
  410 FORMAT(23X,'ZERO-SEQ')
  420 FORMAT('+',36X,F10.4,2X,F10.4,2X,F10.4,//)
```

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2 OF 2

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1.0 4.5 2.8 2.5 5.0 3.2 2.2 5.6 3.6 4.0 2.0 6.3 1.1 1.8 1.25 1.4 1.6

MICROCOPY RESOLUTION TEST CHART
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430 FORMAT(23X,'POS/NEG-SEQ')
440 FORMAT('+',36X,F10.4,2X,F10.4,2X,F10.4,//)
450 FORMAT(//,41X,'****SCALED SOURCE DATA*****',//)
460 FORMAT(28X,'EMAX',5X,'OMEGA',4X,'A-TIME',4X,'B-TIME',4X,'C-TIME',
   14X,'THETA',//)
470 FORMAT(23X,8F10.4,//)
480 FORMAT(//,40X,'RESISTANCE',9X,'INDUCTANCE',//)
490 FORMAT(23X,'ZERO-SEQ')
500 FORMAT('+',38X,1F10.4,10X,1F10.4,//)
510 FORMAT(23X,'POS/NEG-SEQ')
520 FORMAT('+',38X,1F10.4,10X,1F10.4,//)
530 FORMAT(//,43X,'****SCALED LOAD DATA****',//)
540 FORMAT(39X,'CONDUCTANCE',2X,'GAMMA',2X,'CAPACITANCE',//)
550 FORMAT(23X,'ZERO-SEQ')
560 FORMAT('+',38X,3F10.4,//)
570 FORMAT(23X,'POS/NEG-SEQ')
580 FORMAT('+',38X,3F10.4,//)
590 FORMAT(///,'*********************************************OUTPU
   1T DATA**************************************************',//)
600 FORMAT(///,5X,'TIME=',F6.3)
610 FORMAT(/,6X,'NODE',5X,'VA',8X,'VB',8X,'VC',8X,'IA',8X,'IB',8X,
   1'IC',/)
620 FORMAT(I10,8F10.4)
630 FORMAT(/,'*************************************************************
   1****************************',/)
640 FORMAT(20A4)
650 FORMAT(///,5X,'NODE=',I2)
660 FORMAT(/,5X,'TIME',6X,'VA',8X,'VB',8X,'VC',8X,'IA',8X,'IB',8X,
   1'IC',/)
670 FORMAT(10F10.4)
680 FORMAT(//,19X,'VOLTAGE MAXIMA',//)
690 FORMAT(15X,'VA',9X,'VB',9X,'VC',//)
700 FORMAT(9X,3F10.4)
    STOP
    END

    SUBROUTINE YINVRT(Y,GG,GL,GS,GP,NN,GGS)
    DIMENSION Y(50,50),B(50,50)
    N=NN-1
    DO 10 I=1,NN
    DO 10 J=1,NN
    Y(I,J)=0.
 10 B(I,J)=0.
    Y(1,1)=GG+GGS
    Y(1,2)=-GGS
    Y(2,1)=-GGS
    DO 50 I=2,NN
    DO 50 J=2,NN
    IF(J.NE.I)GO TO 30
    IF(I.NE.2)GO TO 20
    Y(I,J)=Y(I,J)+GGS+GS
    GO TO 50
 20 IF(I.NE.NN)GO TO 40
    Y(I,J)=Y(I,J)+GS+GP+GL
    GO TO 50
 30 IF(J.NE.I+1.AND.J.NE.I-1)GO TO 50
    Y(I,J)=Y(I,J)-GS
    GO TO 50
```

```
   40 Y(I,J)=Y(I,J)+2.*GS+GP
   50 CONTINUE
C****INVERT Y-MATRIX****
      DO 60 I=1,NN
   60 B(I,I)=1.
      DO 70 I=1,N
      RATIO=-Y(I+1,I)/Y(I,I)
      Y(I+1,I+1)=Y(I+1,I+1)+RATIO*Y(I,I+1)
      DO 70 J=1,NN
      IF(ABS(RATIO*B(I,J)).LT.1E-10)GO TO 70
      B(I+1,J)=B(I+1,J)+RATIO*B(I,J)
   70 CONTINUE
      DO 80 I=1,N
      K=NN-I
      DO 80 J=1,NN
      RATIO=-Y(K,K+1)/Y(K+1,K+1)
      IF(ABS(RATIO*B(K+1,J)).LT.1E-10)GO TO 80
      B(K,J)=B(K,J)+RATIO*B(K+1,J)
   80 CONTINUE
      DO 90 I=1,NN
      DO 90 J=1,NN
   90 B(I,J)=B(I,J)/Y(I,I)
      DO 110 I=1,NN
      DO 110 J=1,NN
      IF(ABS(B(I,J)).LT.1E-10)GO TO 100
      Y(I,J)=B(I,J)
      GO TO 110
  100 Y(I,J)=0.0
  110 CONTINUE
      RETURN
      END

      SUBROUTINE VXCM(V,Y,CM,N)
      DIMENSION V(50),Y(50,50),CM(50)
      DO 10 J=1,N
      V(J)=0.
      DO 10 K=1,N
      IF(ABS(Y(J,K)*CM(K)).LT.1E-10)GO TO 10
      V(J)=V(J)+Y(J,K)*CM(K)
   10 CONTINUE
      RETURN
      END

      SUBROUTINE MOTOPH(VA,VB,VC,IBA,IBB,IBC,N)
      REAL VA(50),VB(50),VC(50),IBA(50),IBC(50),IBB(50)
      DIMENSION PVA(50),PVB(50),PVC(50),PIBA(50),PIBB(50),PIBC(50)
      DO 10 J=1,N
      PVA(J)=VA(J)+VB(J)+VC(J)
      PIBA(J)=IBA(J)+IBB(J)+IBC(J)
      PVB(J)=VA(J)-2.*VB(J)+VC(J)
      PIBB(J)=IBA(J)-2.*IBB(J)+IBC(J)
      PVC(J)=VA(J)+VB(J)-2.*VC(J)
   10 PIBC(J)=IBA(J)+IBB(J)-2.*IBC(J)
      DO 20 J=1,N
      VA(J)=PVA(J)
      VB(J)=PVB(J)
      VC(J)=PVC(J)
```

```
      IBA(J)=PIBA(J)
      IBB(J)=PIBB(J)
   20 IBC(J)=PIBC(J)
      RETURN
      END
```

```
/GO

/DATA

/DATA
```